Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 9, 2003*
Accepted: *November 21, 2003*

(Anti)symmetric matter and superpotentials from IIB orientifolds

Karl Landsteiner
Instituto de Fisica Theorica C-XVI, Universidad Autonoma de Madrid
28049 Madrid, Spain
E-mail: `KarlLandsteiner@uam.es`

Calin Iuliu Lazaroiu
Humboldt Universität zu Berlin
Newtonstrasse 15, 12489 Berlin-Adlershof, Germany
E-mail: `calin@physik.hu-berlin.de`

Radu Tatar
Theoretical Physics Group, Lawrence Berkeley National Laboratory
Berkeley, CA 94720, U.S.A.
E-mail: `rtatar@socrates.berkeley.edu`

Abstract: We study the IIB engineering of $\mathcal{N} = 1$ gauge theories with unitary gauge group and matter in the adjoint and (anti)symmetric representations. We show that such theories can be obtained as $\mathbb{Z}_2$ orientifolds of certain Calabi-Yau A_2 fibrations, and discuss the explicit T-duality transformation to an orientifolded Hanany-Witten construction. The low energy dynamics is described by a geometric transition of the orientifolded background. Unlike previously studied cases, we show that the orientifold 5-'plane' survives the transition, thus bringing a nontrivial contribution to the effective superpotential. We extract this contribution by using matrix model results and compare with geometric data. A Higgs branch of our models recovers the engineering of SO/Sp theories with adjoint matter through an O5-'plane' T-dual to an O6-plane. We show that the superpotential agrees with that produced by engineering through an O5-'plane' dual to an O4-plane, even though the orientifold of this second construction is replaced by fluxes after the transition.

Keywords: Superstring Vacua, Supersymmetry and Duality, Matrix Models.

http://jhep.sissa.it/archive/papers/jhep112003044/jhep112003044.pdf

JHEP11(2003)044

Contents

1. Introduction

D-brane physics allows for a description of supersymmetric gauge theories leading to novel insights into strong coupling behavior. A particularly fruitful approach in this regard is afforded by geometric engineering. This leads one to consider D-branes partially wrapped on cycles of a nontrivial geometry, which realizes the supersymmetric gauge theory of interest on the uncompactified remnant of the branes' worldvolume.

A prominent example is given by certain local conifold geometries which implement the "large N geometric transitions" [1]–[4]. The starting point of this construction is a IIB background whose closed string sector is described by a singular ADE fibration X_0 over a complex plane parameterized by z. The ADE fiber of X_0 degenerates above certain points of the plane, where the total space acquires conifold singularities. The small resolution $\hat{X}$ of X_0 contains a set of holomorphically embedded two-spheres on which one can wrap D5-branes. Wrapping N_i D5-branes on the i-th exceptional $\mathbb{P}^1$ leads to a four-dimensional $\mathcal{N}=1$ quiver gauge theory on the uncompactified part of the branes' worldvolume. Such theories can be viewed as softly broken $\mathcal{N}=2$ quiver gauge theories. The partial supersymmetry breaking is induced by a superpotential for those chiral multiplets which transform in the adjoint representation of the gauge group. The precise form of the superpotential is determined by the fibration data.

With a nontrivial superpotential, such theories confine at low energies and lead to gaugino condensation. In the geometric realization, this corresponds to a transition in which the exceptional $\mathbb{P}^1$'s shrink to zero size and the resolved geometry is replaced by a 'log-normalizable' smoothing X of the singular fibration X_0. Thus each exceptional $\mathbb{P}^1$ is replaced by a 3-sphere. In this process the D5-branes disappear but their RR-flux is still present and supported on the three-cycles of the deformed geometry. This represents a type-IIB background with nontrivial 3-form fluxes and therefore leads to a superpotential of the form [5]:

$$W_{\text{eff}} = \int H \wedge \Omega = \sum_i \left[N_i \frac{\partial F_0}{\partial S_i} + 2\pi i \alpha_i S_i \right], \tag{1.1}$$

where Ω is the holomorphic three-form of the deformed Calabi-Yau X, H is the type-IIB three form field strength, S_i are the gaugino condensates, N_i the RR-fluxes and α_i the gauge couplings (which are identified with the NS part of the H-flux). Here F_0 is the prepotential of the closed string sector. More precisely, one has:

$$2\pi i S_i = \int_{A_i} \Omega\,, \qquad \frac{\partial F_0}{\partial S_i} = \Pi_i := \int_{B_i} \Omega \tag{1.2}$$

$$N_i = \int_{A_i} H\,, \qquad \alpha_i = \int_{B_i} H\,, \tag{1.3}$$

where A_i, B_i give a symplectic basis of a nonstandard version of $H^3(X, \mathbb{Z})$ (note that the B 'cycles' are non-compact and must be regularized[1]). Many examples of this construction have been studied in detail in recent years [3, 4, 7], including the addition of certain types of orientifolds [8, 9] as well as of fundamental matter which leads to mesonic branches [10] and baryons [11]. A T-dual approach to such transitions was developed in [12, 13], where relations (1.2) appear naturally by studying the low energy dynamics of an M5 brane in MQCD. The MQCD approach [14] is based on lifting a T-dual type-IIA configuration with NS5 branes and D4 branes to M-theory.

The purpose of the present paper is to discuss a modification of the geometric transitions of [1]–[4] which allows us to describe the low energy dynamics of $\mathcal{N}=1$ U(N) gauge theories with one adjoint chiral multiplet and two additional chiral multiplets transforming in either the symmetric or antisymmetric two-tensor representation and its conjugate. The

[1] A way to work with *standard* compact cohomology is provided by the geometric regularization discussed in [6].

tree-level superpotential will have the form:

$$W_{\text{tree}} = \text{tr}\left[W(\Phi) + \tilde{Q}\Phi Q\right] \tag{1.4}$$

where Φ is the adjoint chiral superfield while the symmetric/antisymmetric chiral superfields Q and $\tilde{Q}$ transform as $Q \to UQU^T$ and $\tilde{Q} \to U^*\tilde{Q}U^\dagger$. These fields obey $Q^T = sQ$ and $\tilde{Q}^T = s\tilde{Q}$, where $s = \pm 1$. Here $W(\Phi) = \sum_{k=0}^{d} \frac{g_k}{k+1}\,\text{tr}(\Phi^{k+1})$ is a polynomial of degree $d+1$. Such theories can be realized in the type-IIA set-up though orientifolded Hanany-Witten constructions [15, 16] and have recently been reconsidered in the context of the Dijkgraaf-Vafa conjecture [17, 18, 19]. The IIB approach discussed in this paper will allow us to give the geometric engineering of such theories, which has been missing until now.

As we shall see below, the IIB realization of such systems requires an A_2 fibration together with a $\mathbb{Z}_2$ orientifold. Unlike the cases studied in [22] (which allow one to engineer the SO(N) and Sp($N/2$) gauge theories with one adjoint chiral multiplet), the orientifold action we consider leads to an orientifold five-'plane' which *survives* the large N transition. This gives a string-theoretic explanation of the subtle behavior of such theories which was extracted in [17] and [18, 19] in the context of the matrix-model conjecture of Dijkgraaf and Vafa [23, 24, 25] as well as through the method of generalized Konishi anomalies [26].

The paper is organized as follows. In section 2 we geometrically engineer this class of gauge theories. The crucial ingredient is the addition of an orientifold 5-'plane' to the resolution of a nontrivial A_2 fibration. We also discuss explicitly the T-duality which maps our geometry to the orientifolded Hanany-Witten construction [20], thus explaining the relation between our IIB construction and the IIA description of [15, 16].

Section 3 considers the geometric transition for such models. Since our O5 'plane' survives the transition, it will contribute to the effective superpotential. We extract a geometric expression for this contribution by using the matrix model results of [17] and compare with the proposal of [34].

Section 4 gives a comparative treatment of theories with orthogonal or symplectic gauge group and adjoint matter, which can be engineered as orientifolds of A_1 fibrations by using two different choices of O5 'planes'. The first construction uses an O5-'plane' which is T-dual to an O4-plane, and corresponds to the engineering considered in [22]. The internal part of this 5-'plane' is a compact exceptional curve (a $\mathbb{P}^1$) of the resolved fibration. The second construction uses an O5-'plane' whose internal part is a noncompact curve. This is T-dual to an O6-plane in the Hanany-Witten construction. After the geometric transition, the first construction leads to a pure flux background, with the orientifold being replaced by a contribution to the R-R flux. For the second construction, the O5-'plane' survives the transition and thus contributes to the effective superpotential.

Since both constructions engineer the same field theory, the effective superpotentials must agree. We check this agreement by showing that the second construction can be obtained by considering a certain Higgs branch of the U(N) field theory with symmetric or antisymmetric matter, which allows us to extract the flux-orientifold superpotential by using the results of [17]. This allows us to show that the spectral curves of the associated matrix models agree between the two constructions. Our conclusions are summarized in section 5.

In appendix A, we turn off the superpotential for the adjoint chiral multiplet. We describe the orientifold of the toric resolution of the A_2 singularity and show how the resulting O5 'plane' indeed gives rise to matter in the symmetric or antisymmetric representation.

2. Geometric engineering with a tree-level superpotential

We start by discussing the geometric engineering of our field theory as an orientifold of a type-IIB background with D5-branes. Without the orientifold, this coincides with the background used to engineer the $\mathcal{N} = 1$ A_2 quiver field theory [3, 4] and we start by recalling the latter.

2.1 Geometric engineering of the A_2 quiver theory

2.1.1 The IIB background

Let us consider IIB string theory on the resolution $\hat{X}$ of a non-compact Calabi-Yau threefold X_0 given by a singular A_2 fibration over the complex plane. The background includes a collection of D5-branes wrapping the exceptional $\mathbb{P}^1$'s of the resolution.

Explicitly, the singular space X_0 can be realized as the hypersurface:

$$xy = (u - t_0(z))(u - t_1(z))(u - t_2(z))\,, \tag{2.1}$$

where x, y, u, z are the affine coordinates of $\mathbb{C}^4$ and the polynomials $t_j(z)$ are given by:

$$\begin{aligned} t_0(z) &:= -\frac{2W_1'(z) + W_2'(z)}{3} \\ t_1(z) &:= \frac{2W_2'(z) + W_1'(z)}{3} \\ t_2(z) &:= -t_0(z) - t_1(z) = \frac{W_1'(z) - W_2'(z)}{3}\,. \end{aligned} \tag{2.2}$$

Generically, the affine variety (2.1) has A_1 singularities at $x = y = 0$ and z equal to one of the double points of the planar algebraic curve:

$$\Sigma_0 : \quad (u - t_0(z))(u - t_1(z))(u - t_2(z)) = 0\,. \tag{2.3}$$

This curve gives a reducible 3-section of the A_2 fibration (2.1) whose three rational components C_j are given by $u = t_j(z)$. The double points of Σ_0 sit at the intersection of two such components. These are obtained when $u = t_i(z) = t_j(z)$ for $i \neq j$, which gives the equations:

$$\begin{aligned} t_0(z) - t_2(z) &= -W_1'(z) = 0\,, & u &= -\frac{W_2'(z)}{3} \\ t_1(z) - t_2(z) &= +W_2'(z) = 0\,, & u &= \frac{W_1'(z)}{3} \\ t_0(z) - t_1(z) &= -W_1'(z) - W_2'(z) = 0\,, & u &= -\frac{W_1'(z)}{3} = \frac{W_2'(z)}{3}\,. \end{aligned} \tag{2.4}$$

We let $z_j^{(\alpha)}$ be the roots of $W'_\alpha(z)$, $\tilde{z}_j$ the roots of $W'_1(z)+W'_2(z)$, and denote the corresponding exceptional $\mathbb{P}^1$'s of the resolution $\hat{X}$ by $D_j^{(\alpha)}$ and $\tilde{D}_j$ respectively. Throughout the rest of section 2, we assume that $W'_1(z)$ and $W'_2(z)$ have no common zeroes (which is the generic situation). This means that there is no point in the (z,u)-plane where all three components C_j intersect, i.e. the curve Σ_0 does not have any *triple* points. With this assumption, the sets $\{z_j^{(1)}\}$, $\{z_j^{(2)}\}$ and $\{\tilde{z}_j\}$ are mutually disjoint. In particular, we have $W'_\alpha(z_j^{(\beta)}) \neq 0$ for $\alpha \neq \beta$ and $W'_\alpha(\tilde{z}_j) \neq 0$. Thus the singularities of X_0 are ordinary double points of the fibers $X_0(z_j^{(\alpha)})$ and $X_0(\tilde{z}_j)$. Then $\hat{X}$ is obtained by blowing up each of these double points, thus replacing the singular fibers with their minimal resolutions $\hat{X}(z_j^{(\alpha)})$ and $\hat{X}(\tilde{z}_j)$.

The resolved space $\hat{X}$ can be described explicitly as follows[2] [7]. Consider two copies of $\mathbb{P}^1$ with homogeneous coordinates $[\alpha_j,\beta_j]$ and local affine coordinates $\xi_j := \alpha_j/\beta_j$ (where $j=1,2$). Then $\hat{X}$ is realized as the codimension three subspace in $\mathbb{P}^1[\alpha_1,\beta_1]\times\mathbb{P}^1[\alpha_2,\beta_2]\times\mathbb{C}^4[z,u,x,y]$ cut by the equations:

$$\begin{aligned} \beta_1(u-t_0(z)) &= \alpha_1 x \\ \alpha_2(u-t_1(z)) &= \beta_2 y \\ \alpha_1\beta_2(u-t_2(z)) &= \beta_1\alpha_2\,, \\ (u-t_0(z))(u-t_1(z))(u-t_2(z)) &= xy\,. \end{aligned} \tag{2.5}$$

The map τ which forgets the coordinates on the two $\mathbb{P}^1$ factors implements the resolution. The exceptional $\mathbb{P}^1$'s are simply those fibers of τ which sit above the double points of X_0. One easily checks that $D_j^{(1)}$ is the factor $\mathbb{P}^1[\alpha_1,\beta_1]$ sitting above the double point determined by $z_j^{(1)}$, $D_j^{(2)}$ is the factor $\mathbb{P}^1[\alpha_2,\beta_2]$ sitting above the double point of X_0 determined by $z_j^{(2)}$ and $\tilde{D}_j$ is a 'diagonal' $\mathbb{P}^1$ in $\mathbb{P}^1[\alpha_1,\beta_1]\times\mathbb{P}^1[\alpha_2,\beta_2]$, which sits above the double point determined by $\tilde{z}_j$. More precisely, the exceptional curves are given by the equations:

$$\begin{aligned} &D_j^{(1)}: \quad x=y=0\,, \quad z=z_j^{(1)}\,, \quad u=-\frac{W'_2(z_j^{(1)})}{3}\,, \quad \xi_2=0 \\ &D_j^{(2)}: \quad x=y=0\,, \quad z=z_j^{(2)}\,, \quad u=+\frac{W'_1(z_j^{(2)})}{3}\,, \quad \xi_1=\infty \\ &\tilde{D}_j: \quad x=y=0\,, \quad z=\tilde{z}\,, \quad u=-\frac{W'_1(\tilde{z}_j)}{3}\,, \quad \xi_2=-W'_1(\tilde{z}_j)\xi_1\,. \end{aligned} \tag{2.6}$$

Note that we can use $\xi := \xi_1$ as local affine coordinate on $\tilde{D}_j$ (remember that $W'_1(\tilde{z}_j)$ does not vanish, due to our genericity assumption).

As explained in [7] (and recalled below), the Hanany-Witten construction arises upon performing T-duality with respect to the following U(1) action on $\hat{X}$, which we denote by $\hat{\rho}$:

$$([\alpha_1,\beta_1],[\alpha_2,\beta_2],z,u,x,y) \xrightarrow{\hat{\rho}(\theta)} ([e^{-i\theta}\alpha_1,\beta_1],[\alpha_2,e^{i\theta}\beta_2],z,u,e^{i\theta}x,e^{-i\theta}y)\,. \tag{2.7}$$

[2]Our coordinates differ from those of [7] by the shift $u \to u - t_0(z)$.

This projects as follows on the singular space X_0:

$$(z,\ u,\ x,\ y) \stackrel{\rho_0(\theta)}{\longrightarrow} (z,\ u,\ e^{i\theta}x,\ e^{-i\theta}y)\,. \tag{2.8}$$

The fixed point set of the projected action ρ_0 coincides with the 3-section Σ_0 given in (2.3), while the fixed point locus of (2.7) is its proper transform. The latter has three *disjoint* components $\hat{C}_j$ which are the proper transforms of the components C_j of Σ_0 (in particular $\tau(\hat{C}_j) = C_j$):

$$\begin{aligned}
\hat{C}_0 :&\quad x = y = u - t_0(z) = 0\,, &\quad \alpha_1 &= \alpha_2 = 0\\
\hat{C}_1 :&\quad x = y = u - t_1(z) = 0\,, &\quad \beta_1 &= \beta_2 = 0\\
\hat{C}_2 :&\quad x = y = u - t_2(z) = 0\,, &\quad \beta_1 &= \alpha_2 = 0\,.
\end{aligned} \tag{2.9}$$

It is easy to see that $D_j^{(1)}$ touches each of $\hat{C}_0$ and $\hat{C}_2$ at a single point, while $D_j^{(2)}$ has the same behavior with respect to $\hat{C}_1$ and $\hat{C}_2$. Finally, $\tilde{D}_j$ touches each of $\hat{C}_0$ and $\hat{C}_1$ at a single point.

2.1.2 Local description and relation to the Hanany-Witten construction

As recalled in the introduction, the A_2 quiver theory can also be obtained through a Hanany-Witten construction which involves a flat IIA background containing three types of stacks of $D4$-branes stretching between three NS5-branes. The relation of this construction to the geometric engineering given above is implemented by T-duality, as discussed in a more general context in [7]. To see this explicitly, one considers a local model $\tilde{X} \subset \hat{X}$ of the resolution, which is obtained by gluing three copies U_j $(j = 0, \ldots, 2)$ of $\mathbb{C}^3$ (with affine coordinates x_j, u_j, z_j) according to the identifications:

$$(x_1,\ u_1,\ z_1) = \left(\frac{1}{u_0},\ x_0u_0^2 - W_1'(z_0)u_0,\ z_0\right) \tag{2.10}$$

and:

$$(x_2,\ u_2,\ z_2) = \left(\frac{1}{u_1},\ x_1u_1^2 - W_2'(z_1)u_1,\ z_1\right)\,. \tag{2.11}$$

Then the restricted projection $\tau : \tilde{X} \to X$ is given by:

$$\begin{aligned}
(z;u;x;y) &= \Big(z_0;\ \ x_0u_0 + t_0(z_0);\ \ x_0;\ \ u_0[x_0u_0 - W_1'(z_0)][x_0u_0 - W_1'(z_0) - W_2'(z_0)]\Big)\\
(z;u;x;y) &= \Big(z_1;\ \ x_1u_1 + t_2(z_1);\ \ x_1[x_1u_1 + W_1'(z_1)];\ \ u_1[x_1u_1 - W_2'(z_1)]\Big)\\
(z;u;x;y) &= \Big(z_2;\ \ x_2u_2 + t_1(z_2);\ \ x_2[x_2u_2 + W_2'(z_2)][x_2u_2 + W_1'(z_2) + W_2'(z_2)];\ \ u_2\Big)\,.
\end{aligned} \tag{2.12}$$

In this presentation, it is easy to describe only the exceptional curves $D_j^{(\alpha)}$. Namely $D_j^{(1)}$ is given by the equations $z_1 = z_j^{(1)}$, $u_1 = 0$, while $D_j^{(2)}$ is given by $z_1 = z_j^{(2)}$, $x_1 = 0$.

The Hanany-Witten construction is recovered [7] by T-duality with respect to the U(1) action (2.7), which has the following form in local coordinates:

$$(z_j,\ u_j,\ x_j) \stackrel{\hat{\rho}(\theta)}{\longrightarrow} (z_j,\ e^{-i\theta}u_j,\ e^{i\theta}x_j)\,. \tag{2.13}$$

The fixed point set of this action is given by the curves $\hat{C}_j$, whose local equations are:

$$\hat{C}_0: \quad u_0 = x_0 = 0\,, \qquad \hat{C}_1: \quad u_2 = x_2 = 0\,, \qquad \hat{C}_2: \quad u_1 = x_1 = 0\,. \tag{2.14}$$

The action (2.13) clearly stabilizes the exceptional curves $D_j^{(\alpha)}$. Under T-duality, the loci $\hat{C}_j$ become three NS5 branes denoted $\mathcal{N}_j$, while the D5-branes wrapping $D_j^{(\alpha)}$ are mapped into two stacks of D4 branes, denoted $\mathcal{D}_j^{(\alpha)}$. With our indexing, $\mathcal{N}_2$ is the central five-brane, while $\mathcal{N}_0$ and $\mathcal{N}_1$ are the 'outer' five-branes. The intersections discussed after equations (2.9) show that $\mathcal{D}_j^{(1)}$ stretch between $\mathcal{N}_2$ and $\mathcal{N}_0$, while $\mathcal{D}_j^{(2)}$ stretch between $\mathcal{N}_2$ and $\mathcal{N}_1$. The D5-branes wrapping $\tilde{D}_j$ are mapped into $D4$-branes $\tilde{\mathcal{D}}_j$ stretching between $\mathcal{N}_0$ and $\mathcal{N}_1$ (this is possible because the outer NS5 branes are curved and tilted). This recovers the Hanany-Witten construction of the A_2 quiver theory.

It will be important for our purpose to know the explicit relation between the flat space coordinates of the T-dual type-IIA description and the complex coordinates x_1, u_1, z_1 used in the type-IIB formulation (the relation to the other coordinates x_j, u_j, z_j follows trivially by gluing). Since we don't know the explicit metric on the minimal resolution, we cannot identify the metric data, but we can us a trick[3] to determine a good set of coordinates up to scale factors.

For this, let us combine x_1 and u_1 into a quaternion coordinate $X := x_1 + \mathbf{j}u_1$, where $\mathbf{j}$ is the second quaternion imaginary unit. Then (2.13) becomes the standard U(1) action:

$$X \longrightarrow e^{i\theta} X\,. \tag{2.15}$$

The associated hyperkahler moment map $\vec{\mu} : \mathbb{C}^2[x_1, u_1] \longrightarrow \mathbb{R}^3$ gives a fibration of $\mathbb{C}^2[x_1, u_1]$ over $\mathbb{R}^3$ whose generic fiber is a circle (the fiber collapses to a point precisely above the origin of $\mathbb{R}^3$). We shall denote the cartesian coordinates of the $\mathbb{R}^3$ base by x^4, x^5, x^6 and let x^7 be the (periodic) coordinate along the S^1 fiber. Then $x^4 + ix^5$ gives the complex part of the hyperkahler moment map, while x^6 is its real part:

$$x^4 + ix^5 = x_1 u_1\,, \qquad x^6 = \frac{1}{2}(|x_1|^2 - |u_1|^2)\,. \tag{2.16}$$

The dual type-IIA description is realized in the Minkowski space $\mathbb{R}^{1,9}$ with coordinates $x^0, \ldots, x^9$, where the 'internal' coordinates $x^4, \ldots, x^9$ are related to x_1, u_1, z_1 through equation (2.16) and:

$$x^8 + ix^9 = z\,. \tag{2.17}$$

It is now easy to see that the NS5-brane worldvolumes extend along the directions $x^0, \ldots, x^3$ and x^8, x^9, while being localized at $x^7 = 0$. The central NS5-brane $\mathcal{N}_2$ is located at $x^4 = x^5 = x^6 = x^7 = 0$ and extends in the directions x^8 and x^9. The other two NS5-branes

[3] It is non-trivial to give a full justification of the identification (2.16) used below. This is because in the presence of a superpotential the metric on the resolution need not be compatible with the hyperkahler structure we introduce for x_1, u_1. However, the situation is somewhat similar to that of [27] and we expect that an argument along the lines of that paper can be applied to our situation. Another approach would be to apply the Buscher formulas [28] to a appropriate supergravity solution for intersecting NS branes and D4 branes. In any case, the coordinate identification (2.16) does reproduce the expected physics.

sit at $x^6 = \pm\infty, x^7 = 0$ and are curved in the x^4, x^5, x^8, x^9 directions according to the equations:

$$x^4 + ix^5 = -W_1'(z)\,, \qquad x^6 = +\infty \qquad \text{for} \qquad \mathcal{N}_0 \tag{2.18}$$

and:

$$x^4 + ix^5 = +W_2'(z)\,, \qquad x^6 = -\infty \qquad \text{for} \qquad \mathcal{N}_1\,. \tag{2.19}$$

The $D4$-brane worldvolumes of $\mathcal{D}_j^{(\alpha)}$ extend along $x^0, \ldots, x^3$ as well as x^6 and are localized[4] at $x^4 = x^5 = x^7 = 0$ and $z = z_j^{(\alpha)}$.

2.2 Adding the orientifold

2.2.1 The IIB description

In the type-IIB set-up, we consider the case when $W_1(z) = W_2(-z) := W(z)$. Then $W_1'(z) = -W_2'(-z) = W'(z)$ and we have:

$$\begin{aligned} t_0(z) &= \frac{-2W'(z) + W'(-z)}{3} := t(z) \\ t_1(z) &= \frac{-2W'(-z) + W'(z)}{3} = t(-z) \\ t_2(z) &= \frac{W'(z) + W'(-z)}{3} = -t(z) - t(-z)\,. \end{aligned} \tag{2.20}$$

In this situation, we can index the points $z_j^{(\alpha)}$ such that $z_j^{(1)} = -z_j^{(2)} := z_j^+$, and we let $z_j^- := z_j^{(2)}$. Then $z_j^- = -z_j^+$ and z_j^+ are the roots of $W'(z)$. We also index the points $\tilde{z}_j$ (which are the roots of the polynomial $W'(z) - W'(-z)$) by positive, zero and negative integers j such that $\tilde{z}_{-j} = -\tilde{z}_j$ (and in particular $\tilde{z}_0 = 0$). With these conventions, the exceptional curves $D_j^{(1)}$ and $D_j^{(2)}$ will be denoted by D_j^+ and D_j^-; these are distributed in symmetric pairs with respect to the origin of the z-plane. The curves $\tilde{D}_j$ are also distributed in symmetric pairs $(\tilde{D}_j, \tilde{D}_{-j})$, except for the central curve $\tilde{D}_0$ sitting above $z = 0$. By our genericity assumption, each of these exceptional curves sits in a distinct fiber of $\hat{X}$ over the z-plane.

When (2.20) are satisfied, the resolution $\hat{X}$ admits a $\mathbb{Z}_2$ symmetry $\hat{\kappa}$ given by:

$$([\alpha_1, \beta_1], [\alpha_2, \beta_2], z, u, x, y) \xrightarrow{\hat{\kappa}} ([-\beta_2, \alpha_2], [-\beta_1, \alpha_1], -z, u, -y, -x) \tag{2.21}$$

which acts as follows on the affine coordinates $\xi_j = \alpha_j/\beta_j$ of the two $\mathbb{P}^1$ factors:

$$\xi_1 \longleftrightarrow -\frac{1}{\xi_2} \tag{2.22}$$

and projects to the following involution κ_0 of X_0:

$$(z,\ x,\ y,\ u) \xrightarrow{\kappa_0} (-z,\ -y,\ -x,\ u)\,. \tag{2.23}$$

[4] Localization in the direction x^7 is due to the fact that there is no B_{NS} flux through the exceptional $\mathbb{P}^1$'s.

The action (2.21) stabilizes the central fiber $\hat{X}(0)$, while (2.23) stabilizes $X_0(0)$. Tracing through the equations, we find that the symmetry interchanges D_j^+ and D_j^-, while mapping $\tilde{D}_j$ into $\tilde{D}_{-j}$. In particular, the $\mathbb{Z}_2$ action stabilizes $\tilde{D}_0$, on which it acts as:

$$\xi \longrightarrow \frac{1}{W'(0)}\xi^{-1} . \tag{2.24}$$

This fixes two points $p_\pm$ of $\tilde{D}_0$, given by the roots $\xi_\pm = \pm W'(0)^{-1/2}$.

The action (2.21) maps $\hat{C}_0$ into $\hat{C}_1$ and stabilizes $\hat{C}_2$ while fixing the following locus in $\hat{X}$:

$$\hat{O}: \quad y = -x\,, \quad z = 0\,, \quad \xi_1\xi_2 = -1\,. \tag{2.25}$$

Choosing ξ_2, u and x as local coordinates, $\hat{O}$ can be described by the equations:

$$\begin{aligned} \xi_2\left(u + \frac{W'(0)}{3}\right) &= -x \\ u - \frac{2W'(0)}{3} &= -\xi_2^2 \end{aligned} \tag{2.26}$$

This is a smooth rational curve $x = \xi_2(\xi_2^2 - W'(0))$ (parameterized by ξ_2) which sits in the fiber $\hat{X}(0)$ above the point $z = 0$. Its projection is the fixed point locus of the action (2.23):

$$O_0: \quad z = 0\,, \quad x = -y\,, \quad x^2 + \left(u + \frac{W'(0)}{3}\right)^2\left(u - \frac{2W'(0)}{3}\right) = 0\,, \tag{2.27}$$

which is a nodal curve sitting in $X_0(0)$. In fact, $\hat{O}$ is the proper transform of O_0 under the blow-up $\hat{X}(0) \to X_0(0)$ (the singular point $(x,u) = (0, -\frac{W'(0)}{3})$ is replaced by $(x, u, \xi_2) = (0, -\frac{W'(0)}{3}, \pm W'(0)^{1/2})$).

We shall use $\hat{\kappa}$ as an orientifold action on our type-IIB theory. Thus our background will contain an O5-'plane', whose worldvolume spans the directions $x^0, \ldots, x^3$ and the rational curve (2.25).

2.2.2 Relation to the orientifolded brane construction in IIA

To understand the T-dual orientifold, we start with the local description of the involution $\hat{\kappa}$ in the patches U_j:

$$\begin{aligned} (z_1,\ x_1,\ u_1) &\xrightarrow{\hat{\kappa}} (-z_1,\ -u_1,\ -x_1) \\ (z_0,\ x_0,\ u_0) &\xrightarrow{\hat{\kappa}} (-z_2,\ -u_2,\ -x_2)\,, \end{aligned} \tag{2.28}$$

which gives the following local description of the orientifold 'plane':

$$\hat{O}: \quad z = 0\,, \qquad x_1 = -u_1\,. \tag{2.29}$$

We next note the relation:

$$\hat{\rho}(\theta) \circ \hat{\kappa} = \hat{\kappa} \circ \hat{\rho}(-\theta)\,, \tag{2.30}$$

which shows that the IIB involution inverts the S^1 coordinate x^7. By T-duality, the orientifold 5-'plane' must thus become an orientifold 6-plane, whose equations are easily

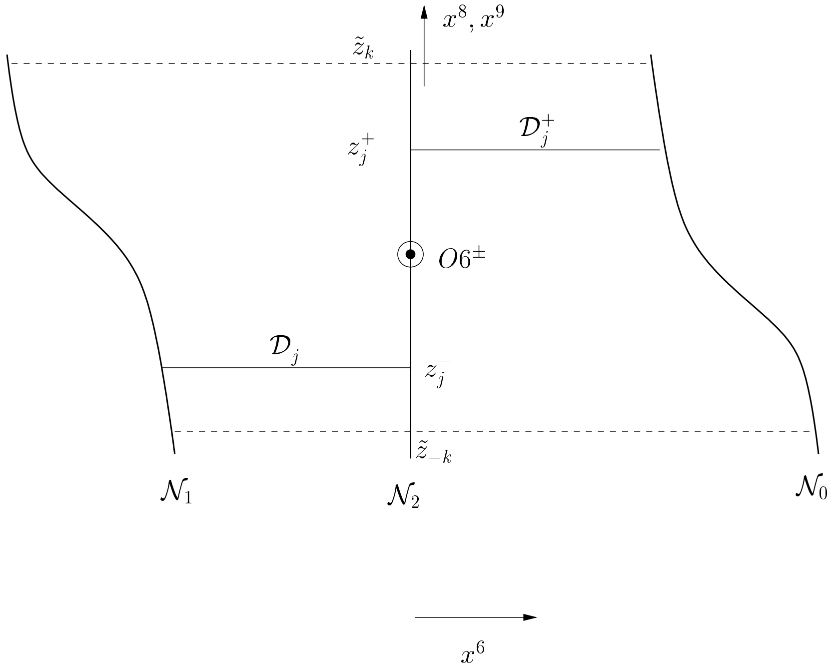

Figure 1: The T-dual brane configuration in IIA on flat $\mathbb{R}^{1,9}$. We show only the coordinates x^6, x^8 and x^9. The coordinates x^4 and x^5 point outside of the figure.

extracted from the coordinate transformations (2.16) and (2.17):

$$x^6 = x^8 = x^9 = 0\,. \tag{2.31}$$

Thus the orientifold extends in the directions x^4, x^5 and x^7 and in particular it intersects orthogonally the central NS5-brane $\mathcal{N}_2$. The IIA involution inverts the sign of x^6, x^8 and x^9, while leaving the other coordinates unchanged (see figure 1). This is precisely the situation considered in [15, 16].[5] The IIA orientifold permutes the the stacks $\mathcal{D}_j^+$ and $\mathcal{D}_j^-$ and the outer NS5 branes $\mathcal{N}_0$ and $\mathcal{N}_1$. It stabilizes the central five-brane $\mathcal{N}_2$ while acting nontrivially on its worldvolume. We stress that the O6-plane is orthogonal to the central NS5 brane as well as to the D4-branes $\mathcal{D}_j^+$ and $\mathcal{D}_j^-$. This is quite different from the situation considered in the papers [29, 30, 31], which discussed an alternate orientifold construction of the same IIA brane configuration. The latter construction involves an O6-plane which *contains* the central NS5 brane and leads to a *chiral* theory containing both symmetric and antisymmetric matter as well as eight fundamentals. The geometric engineering and matrix model relevant for that situation are studied in [32].

2.3 Geometric interpretation of the moduli space

The brane configurations and their T-dual geometries encode information about the moduli space of our field theories. A general discussion of deformations for $\mathcal{N} = 2, A_k$ quiver field theories was developed in [7].

[5] Except that we do not add any 6-branes in our case. Our IIA coordinates are related to those of [16] by the relabeling $(x^4, x^5) \longleftrightarrow (x^8, x^9)$.

Consider first the configuration without the orientifold. In $\mathcal{N}=1$ language, the $\mathcal{N}=2, \mathrm{U}(N_1)\times\mathrm{U}(N_2)$ theory with 2 stacks of N_1 and N_2 D5 branes wrapped on the two exceptional $\mathbb{P}^1$'s of the resolution of an A_2 singularity has the A_2 quiver superpotential:

$$W = \mathrm{tr}_2(Q_{12}\Phi_1 Q_{21}) - \mathrm{tr}_1(Q_{21}\Phi_2 Q_{12}) \tag{2.32}$$

where Φ_j are the adjoint chiral multiplets obtained by decomposing the $\mathcal{N}=2$ vector multiplets and Q_{12} and Q_{21} are chiral multiplet bifundamentals in the representations $(\bar{N}_1, N_2)$ and $(N_1, \bar{N}_2)$ respectively. This is deformed to an $\mathcal{N}=1$ quiver field theory by adding superpotentials $\mathrm{tr}_1 W_1(\Phi_1)$ and $\mathrm{tr}_2 W_2(\Phi_2)$, which corresponds to fibering the A_2 singularity over the z-plane. Here W_α are polynomials of degrees d_α and their derivatives determine the fibration data as in (2.1), (2.2).

In order to determine the $\mathcal{N}=1$ moduli space, one solves the F-term equations and divides by the complexified gauge group [3]. The generic vacuum arises by taking $\Phi_\alpha = \mathrm{diag}(z_1^{(\alpha)}\mathbf{1}_{N_1^{(\alpha)}}\dots z_{d_\alpha}^{(\alpha)}\mathbf{1}_{N_{d_\alpha}^{(\alpha)}}, \tilde{z}_1\mathbf{1}_{\tilde{N}_1}\dots\tilde{z}_k\mathbf{1}_{\tilde{N}_{\tilde{d}}})$ where $z_j^{(\alpha)}$ are roots of W'_α, $\tilde{z}_l$ are the roots of $W'_1(z)+W'_2(z)$ ($\tilde{d}$ is the degree of this latter polynomial) and we have $\sum_j N_j^{(\alpha)} + \sum_l \tilde{N}_l = N_\alpha$. This leads to the residual gauge group:

$$\prod_j \mathrm{U}(N_j^{(1)}) \times \prod_j \mathrm{U}(N_j^{(2)}) \times \prod_l \mathrm{U}(\tilde{N}_l) \tag{2.33}$$

where $\mathrm{U}(N_j^{(\alpha)})$ are embedded in $\mathrm{U}(N_\alpha)$ (corresponding to zero eigenvalues for the bifundamentals) and $\mathrm{U}(\tilde{N}_l)$ are diagonally embedded in $\mathrm{U}(N_1)\times\mathrm{U}(N_2)$ (corresponding to non-zero eigenvalues for the bifundamentals).

In terms of the geometry discussed above, this corresponds to $N_j^{(\alpha)}$ D5 branes wrapped on the curves $D_j^{(\alpha)}$ and $\tilde{N}_l$ D5 branes on the curves $\tilde{D}_l$. The T-dual brane configuration contains several stacks of D4 branes. The stacks $\mathcal{D}_j^{(1)}$ (respectively $\mathcal{D}_j^{(2)}$) contain $N_j^{(1)}$ (respectively $N_j^{(2)}$) D4 branes and stretch from the central NS5 brane $\mathcal{N}_2$ to the left (respectively right) NS5 branes $\mathcal{N}_0$ and $\mathcal{N}_1$. The stacks $\tilde{\mathcal{D}}_j$ contain $\tilde{N}_l$ D4 branes and stretch from the left NS5 brane $\mathcal{N}_0$ to the right NS5 brane $\mathcal{N}_1$.

What happens if one adds an orientifold plane? There are two types of orientifolds in the IIA construction, either an O4 plane parallel to the D4 branes or an O6 plane which is orthogonal to the D4 branes. In the T-dual geometry, the O4 plane becomes an O5 'plane' wrapped on one of the $\mathbb{P}^1$ cycles (this is the configuration used in [22]). On the other hand, the O6 plane can be chosen in at least two different ways, which were discussed in [15, 16] and [29, 30, 31] respectively. The first choice [15, 16] is to take the O6 plane to be orthogonal not only to the D4 branes but also to the central NS5 brane. This leads to the non-chiral theories studied in the present paper. With the second choice [29, 30, 31], the O6 plane is orthogonal to the D4-branes but contains the central NS5 brane, which leads to the chiral theories studied in [32].

With the first choice of O6 plane, the dual IIB orientifold has the action discussed above (see also [33, 8] for related though simpler models). As we saw in the previous subsection, the orientifold symmetry requires $W_1(z) = W_2(-z) := W(z)$, in which case the polynomial $W_1'(z)+W_2'(-z) = W'(z)-W'(-z)$ has degree $2\delta+1$, where $\delta = \left[\frac{d-1}{2}\right]$ (see [17]). The orientifold projection forces a symmetric distribution of the exceptional curves $D_j^{(1)}$ and $D_j^{(2)}$ (now denoted by D_j^+ and D_j^-) and an arrangement of $\tilde{D}_l$ into symmetric pairs $(\tilde{D}_l, \tilde{D}_{-l})$, together with the central curve $\tilde{D}_0$. In the IIA brane construction, this means that we symmetrically identify the $N_j^{(1)}$ D4 branes $\mathcal{D}_j^{(1)} = \mathcal{D}_j^+$ with the $N_j^{(2)}$ D4 branes $\mathcal{D}_j^{(2)} = \mathcal{D}_j^-$. We also identify the D4 branes $\tilde{D}_j$ with $\tilde{D}_{-j}$ which go from the left NS5 brane to the right NS5 brane and are located at opposite positions along the middle NS5 brane, except for the stack $\tilde{\mathcal{D}}_0$ of D4 branes located at $z=0$, which intersects the O6 plane and is mapped to itself under the orientifold action. For these identifications, one enumerates $z_j^{(\alpha)}$ and $\tilde{z}_j$ such that $z_j^{(1)} = z_j^+ = -z_j^- = -z_j^{(2)}$ and $\tilde{z}_{-l} = -\tilde{z}_l$. One also takes $N_1 = N_2 = N$ as well as $N_j^{(1)} = N_j^{(2)}$ and $\tilde{N}_{-l} = \tilde{N}_l$.

As explained in [15, 16] and [17], the orientifold projection on the bifundamental fields produces a symmetric or antisymmetric field and its conjugate (the symmetric field appears if one uses an $O6^+$ plane and the antisymmetric field appears for an $O6^-$ plane). Then, the identified $N_j^{(1)} = N_j^{(2)}$ D4 branes $\mathcal{D}_j^{(1)} \equiv \mathcal{D}_j^{(2)}$ correspond to zero vev for the symmetric/antisymmetric field and nonzero vev for the adjoint field. Since the superpotentials $W_j(\Phi_j)$ correspond to tilting and bending of the outer NS5-branes, these D4 branes are displaced along the middle NS5 brane $\mathcal{N}_2$ and they do not intersect the O6 plane.

The stacks of identified $\tilde{N}_l = \tilde{N}_{-l}$ D4 branes $\tilde{\mathcal{D}}_l \equiv \tilde{\mathcal{D}}_{-l}$ correspond to nonzero vevs for both the adjoint and symmetric/antisymmetric fields. Such D4 branes are displaced with respect to the central NS5 brane and the O6 plane and do not touch any of them, stretching directly between the left and right NS5 branes.

The last stack of D4 branes $\tilde{\mathcal{D}}_0$ corresponds to zero vev for the adjoint field but nonzero vev for the symmetric/antisymmetric field. Such D4 branes are displaced along the O6 plane and touch the left and right NS5 branes, but not the middle NS-brane $\mathcal{N}_2$. Because these D4 branes touch the O6 plane, the projected gauge group is $\mathrm{SO}(N_0)$ or $\mathrm{Sp}(N_0/2)$, depending on whether $s=+1$ or $s=-1$.

This discussion agrees with the results of section 2.1 of [17] and recovers the fact that in the generic supersymmetric vacuum the adjoint field has a vev:

$$\Phi = \mathrm{diag}(0_{\tilde{N}_0}, z_1^+ 1_{N_1} \ldots z_d^+ 1_{N_d}, \tilde{z}_1 1_{\tilde{N}_1}, -\tilde{z}_1 1_{\tilde{N}_1} \ldots \tilde{z}_\delta 1_{\tilde{N}_\delta}, -\tilde{z}_\delta 1_{\tilde{N}_\delta})\,, \tag{2.34}$$

while the residual gauge group is:[6]

$$\prod_{i=1}^{d} \mathrm{U}(N_i) \times \prod_{j=1}^{\delta} \mathrm{U}(\tilde{N}_j) \times G_0 \tag{2.35}$$

where $\sum_{i=1}^d N_i + 2\sum_{l=1}^\delta \tilde{N}_l + \tilde{N}_0 = N$ with $\delta = \left[\frac{d-1}{2}\right]$ and $G_0 = \mathrm{SO}(\tilde{N}_0)$ if $s=+1$, respectively $G_0 = \mathrm{Sp}(\tilde{N}_0/2)$ if $s=-1$.

[6]We use the convention that U(0) is the trivial group.

3. The geometric transition and the effective superpotential

In this section we consider the geometric transition of [1, 2, 3, 4], which replaces the D5 branes by fluxes through the 3- cycles of a deformed geometry. The D5 branes wrapped on the $\mathbb{P}^1$ cycles go through the transition in the usual way [1, 2, 3, 4], so it suffices to concentrate on understanding the orientifold projection after the transition.

3.1 The orientifold after the geometric transition

Let X be the smoothing of the singular threefold X_0, which is described by the equation:

$$xy = u^3 - p(z)u - q(z)\,, \tag{3.1}$$

where (as in [3]) we only consider log-normalizable deformations (the explicit expression of p, q for such deformations is recalled in equation (4.31) of section 4.3.2 below).

After the geometric transition $\hat{X} \longrightarrow X_0 \longrightarrow X$ of [1]–[4], the D5-branes wrapping the exceptional fibers of $\hat{X}$ will be replaced by fluxes through the S^3 cycles created by smoothing. If one starts with a $\mathbb{Z}_2$ symmetric hypersurface X_0 (which is achieved by requiring $W_1(z) = W_2(-z) := W(z)$), then one can restrict the smoothing X by requiring that p and q are even, so that X will admit the $\mathbb{Z}_2$ symmetry (2.23). In this situation, we add the orientifold and ask what happens after the transition.

Since the deformed Calabi-Yau admits the action (2.23), it is clear that the orientifold will survive the transition, being mapped into an orientifold of IIB with geometric action κ given by equations (2.23). The A_2 fibration X admits a multisection Σ (the deformation of Σ_0) given by the equations $x = y = 0$, which imply:

$$\Sigma : \quad u^3 - p(z)u - q(z) = 0\,. \tag{3.2}$$

Let us write this Riemann surface as a triple cover of the z-plane:

$$u^3 - p(z)u - q(z) = (u - u_0(z))(u - u_1(z))(u - u_2(z)) \tag{3.3}$$

where $u_2(z) = -u_0(z) - u_1(z)$ and we index the branches $u_j(z)$ such that they are deformations of $t_j(z)$. Then Σ is invariant under the involution and we have $u_0(-z) = u_1(z)$ and $u_2(-z) = u_2(z)$. The smoothing replaces the double points $z_i^\pm$ and $\tilde{z}_j$ of Σ_0 with cuts of Σ which we denote by $I_{\pm i}$ and $\tilde{I}_j$. The cuts I_i connect the branches u_0 and u_2 if $i > 0$ and the branches u_1 and u_2 if $i < 0$, while each of the cuts $\tilde{I}_j$ connects the branches u_0 and u_1. As discussed in [17], these cuts are distributed symmetrically with respect to the origin of the z-plane, i.e. we have $I_{-i} = -I_i$ and $\tilde{I}_{-j} = -\tilde{I}_j$. In particular, the central cut $\tilde{I}_0$ is symmetric under the point reflection $z \to -z$.

The orientifold action (2.23) on X has fixed locus given by:

$$O : \quad z = 0\,, \qquad x = -y\,, \tag{3.4}$$

which defines a (generically smooth) elliptic curve lying inside the central fiber $X(0)$:

$$x^2 + u^3 - p(0)u - q(0) = 0\,. \tag{3.5}$$

This locus corresponds to an O5-'plane' which survives the geometric transition.

Thus we end up with a compactification with NS-NS and R-R fluxes plus an orientifold five-'plane' whose internal part is given by (3.5). As explained in [34], such a compactification will produce a superpotential which receives contributions from fluxes and from the orientifold fixed locus. Since the geometric transition corresponds to confinement in the low energy field theory on the noncompact part of the D5-brane worldvolumes, the flux-orientifold superpotential after the transition can be identified with the effective superpotential of this theory for the glueball superfields.

3.2 The matrix model prediction for the flux-orientifold superpotential

In order to compute the effective superpotential from geometry, we must calculate the periods of the holomorphic three-form of the deformed Calabi-Yau X. For this, recall from [2, 3, 4] that every one-cycle of the Riemann surface Σ defines a 3-cycle of X obtained by considering a certain S^2 fibration associated with that one-cycle. Accordingly, we consider the following set of three-cycles of X. Let A_i be a three-cycle which is obtained as an S^2 fibration over a one-cycle a_i of Σ which surrounds the cut I_i. We can view A_i as the three-cycle produced by smoothing the singular point of X_0 siting above z_i. According to our considerations in the previous section this corresponds to the $\mathrm{U}(N_i)$ component of the unbroken gauge group. We choose A_i such that the $\mathbb{Z}_2$ symmetry κ maps it into the cycle A_{-i}, and we consider the κ-invariant linear combination A_i+A_{-i} for $i=1,\ldots,d$. Similarly, we consider κ-invariant cycles $\tilde{A}_j+\tilde{A}_{-j}$ for $j=1,\ldots,\delta$, where the 3-cycles $\tilde{A}_j$ correspond to one-cycles $\tilde{a}_j$ of Σ surrounding the branch cut of type $\tilde{I}_j$ and to the component $\mathrm{U}(\tilde{N}_j)$ of the unbroken gauge group. They are produced by smoothing the singular point of X_0 which sits above $\tilde{z}_j$, and are chosen such that $\kappa(\tilde{A}_j)=\tilde{A}_{-j}$. Finally, we consider a three-cycle $\tilde{A}_0$ which is invariant under κ. It corresponds to a one-cycle $\tilde{a}_0$ on Σ which surrounds the cut $\tilde{I}_0$ and arises by smoothing the singular point of X_0 sitting above the origin. This 3-cycle also corresponds to the component $G_0=\mathrm{SO}(\tilde{N}_0)$ or $\mathrm{Sp}(\tilde{N}_0/2)$ of the unbroken gauge group.

We are thus led to consider two classes of A-periods of the holomorphic 3-form. The first class is given by:

$$2\pi i S_i=\frac{1}{2}\int_{A_i+A_{-i}}\Omega=\int_{A_i}\Omega\,, \tag{3.6}$$

where the last equality holds because of the invariance of Ω under κ. We also have the periods:

$$2\pi i\tilde{S}_j=\frac{1}{2}\int_{\tilde{A}_j+\tilde{A}_{-j}}\Omega=\int_{\tilde{A}_j}\Omega\,, \tag{3.7}$$

and

$$2\pi i\tilde{S}_0=\frac{1}{2}\int_{\tilde{A}_0}\Omega\,. \tag{3.8}$$

The cycles which define the periods S_i for $i=1,\ldots,d$ and $\tilde{S}_j$ for $j=1,\ldots,\delta$ will be called *long* invariant cycles, whereas $\tilde{A}_0$ is a *short* invariant cycle.[7]

[7]The notion of long and short invariant cycles plays a prominent role in the theory of boundary singularities [35].

The fluxes of the three-form H are:

$$N_i = \int_{A_i} H\,, \quad i=-d,\ldots,-1,1,\ldots,d$$
$$\tilde{N}_j = \int_{\tilde{A}_j} H\,, \quad j=-\delta,\ldots,\delta\,. \tag{3.9}$$

The collection $A_i, \tilde{A}_j$ can be completed by considering a set of 3-cycles $B_i(\Lambda)$ and $\tilde{B}_j(\tilde{\Lambda})$ with intersection numbers:

$$\langle A_i, B_j\rangle = \delta_{ij}\,, \qquad \text{for } i,j=-d,\ldots,-1,1,\ldots,d$$
$$\langle \tilde{A}_i, \tilde{B}_j\rangle = \delta_{ij}\,, \qquad \text{for } i,j=-\delta,\ldots,\delta \tag{3.10}$$

and zero otherwise.

The B-cycles depend explicitly on two complex cutoffs $\Lambda, \tilde{\Lambda}$ whose absolute values we take to be large. Then $B_i(\Lambda)$ and $\tilde{B}_i(\tilde{\Lambda})$ are constructed as certain S^2 fibrations over one-cycles $b_i(\Lambda)$ and $\tilde{b}_j(\tilde{\Lambda})$ on Σ which are constrained to pass through the points of Σ obtained by lifting Λ and $\tilde{\Lambda}$ from the z-plane. Namely, $b_i(\Lambda)$ connects the lifts of Λ to the branches u_0 and u_2 (if $i>0$) respectively u_1 and u_2 (if $i<0$), while $\tilde{b}_j(\tilde{\Lambda})$ connects the two lifts of $\tilde{\Lambda}$ to the branches u_0 and u_1. Mathematically, this amounts to working with an open-closed (punctured) Riemann surface obtained by removing the lifts of Λ and $\tilde{\Lambda}$ from the original closed Riemann surface Σ.

Thus the remaining periods are given by:

$$\Pi_i = \int_{B_i(\Lambda)} \Omega \qquad \text{for } i=-d,\ldots,-1,1,\ldots,d\,,$$
$$\tilde{\Pi}_j = \int_{\tilde{B}_j(\tilde{\Lambda})} \Omega \qquad \text{for } j=-\delta,\ldots,\delta\,. \tag{3.11}$$

Relations similar to (3.9) hold for the gauge couplings:

$$\alpha_i = \int_{B_i(\Lambda)} H\,, \qquad \text{for } i=1,\ldots,d$$
$$\tilde{\alpha}_j = \int_{\tilde{B}_j(\tilde{\Lambda})} H\,, \qquad \text{for } j=0,\ldots,\delta\,. \tag{3.12}$$

As in [2, 3, 4], the period integrals of Ω can be reduced to corresponding period integrals of the meromorphic one-form $u\frac{dz}{2\pi i}$ on the Riemann surface (3.2) (for this, the normalization of Ω must be chosen appropriately):

$$S_i = \int_{a_i} u\frac{dz}{2\pi i}\,, \qquad \tilde{S}_j = \int_{\tilde{a}_j} u\frac{dz}{2\pi i}\,, \qquad \tilde{S}_0 = \frac{1}{2}\int_{\tilde{a}_0} u\frac{dz}{2\pi i}$$
$$\Pi_i = \int_{b_i(\Lambda)} udz\,, \qquad \tilde{\Pi}_j = \int_{\tilde{b}_j(\tilde{\Lambda})} udz\,, \qquad \tilde{\Pi}_0 = \int_{\tilde{b}_0(\tilde{\Lambda})} udz\,. \tag{3.13}$$

The flux-orientifold superpotential takes the form:

$$W_{\text{eff}} = \sum_{i=1}^{d}\left[N_i\Pi_i + 2\pi i\alpha_i S_i\right] + \sum_{j=1}^{\delta}\left[\tilde{N}_j\tilde{\Pi}_i + 2\pi i\tilde{\alpha}_j\tilde{S}_j\right] + \frac{\tilde{N}_0}{2}\tilde{\Pi}_0 + 2\pi i\tilde{\alpha}_0\tilde{S}_0 + 4F_1\,. \tag{3.14}$$

The first terms arise from the fluxes, while the last term is the orientifold contribution. The factor of 1/2 in front of $\tilde{N}_0\tilde{\Pi}_0$ is due to the fact that $\tilde{N}_0$ as defined in (3.9) arises by integrating H over the short cycle $\tilde{A}_0$. Up to the orientifold contribution F_1, this is precisely half of the flux-superpotential of an A_2 quiver theory with a κ-symmetric arrangement of fluxes and deformations. We have the special geometry relations:

$$\Pi_i = \frac{\partial F_0}{\partial S_i}\,, \qquad \tilde{\Pi}_j = \frac{\partial F_0}{\partial \tilde{S}_j}\,, \tag{3.15}$$

where F_0 is the closed string prepotential. We are now going to use matrix model arguments to show that the orientifold contribution F_1 takes the form:

$$F_1 = -\frac{s}{4}\int_{\tilde{b}_0(\tilde{\Lambda})} u\,dz\,. \tag{3.16}$$

As in [23], the low energy effective superpotential of our field theory is encoded by the dynamics of the topological sector of open strings connecting the B-type branes wrapping the exceptional $\mathbb{P}^1$ cycles of $\hat{X}$. In our case, this two dimensional theory is obtained by reducing the holomorphic Chern-Simons action, similar to the argument given in [24] for the ADE quiver theories, but including the orientifold projection. One easily finds that the open topological sector reduces to the holomorphic [36] matrix model constructed in [17]. This holomorphic matrix model was studied in [17, 36, 18, 19]. As explained in those references, the matrix model leads to a prescription for computing the gaugino superpotential in the $\mathcal{N}=1$ U(N) field theory with one adjoint and one symmetric or antisymmetric chiral multiplet. Through geometric engineering, the latter is realized as the low energy limit of the flux-orientifold compactification on X, and the gaugino superpotential must coincide with the flux-orientifold superpotential (3.14).

As explained in [17], the planar limit of this model's traced resolvent coincides up to a shift with the sheet u_0 of the Riemann surface Σ, which is identified with the matrix model's spectral curve. Following the general prescription of the Dijkgraaf-Vafa correspondence, the closed string prepotential F_0 is realized as the planar limit of the matrix model's free energy (= microcanonical generating function), while the orientifold term F_1 gives the $\mathbb{RP}^2$ contributions to the matrix model free energy. In particular, the orientifold term F_1 can be expressed in terms of matrix model data. It was furthermore proven in [37] that the $\mathbb{RP}^2$ part of the matrix model partition function contributes with a relative factor of 4 to the gluino superpotential (3.14). Notice that the D5 brane charge of our orientifold is given by $-s$. Therefore combining (3.16) and (3.14) it turns out that the orientifold contribution to the superpotential is proportional to its D5 brane charge.

We shall use the results of [17] in order to extract the geometric description of F_1 given in (3.16).

Recall from [17] that the exact loop equations for the traced matrix model resolvent $\omega(z) = \mathrm{tr}(1/z - M)$ (where M is the matrix associated with Φ) have the form:

$$\langle \omega(z)^2 + \omega(z)\omega(-z) + \omega(-z)^2\rangle = \int_\gamma \frac{dx}{2\pi i}\frac{2xU'(x)}{z^2-x^2}\langle\omega(x)\rangle\,, \tag{3.17}$$

$$\langle \omega(z)^2\omega(-z) + \omega(z)\omega(-z)^2\rangle - \frac{1}{N^2}\frac{\langle\omega(z)+\omega(-z)-2\omega(0)\rangle}{4z^2} = \int_\gamma \frac{dx}{2\pi i}\frac{2xU'(x)}{z^2-x^2}\langle\omega(x)\omega(-x)\rangle\,.$$

where $U(z) = W(z) + (t_{-1} + \frac{s}{2N}) \ln(z)$ and γ is a contour encircling the poles of $\omega(z)$ but not the point z nor the poles of $\omega(-z)$. By expanding these loop equations it was shown in [17] that the contribution of $\mathbb{RP}^2$ diagrams to the matrix model free energy is given by:

$$F_1 = \frac{s}{2}\frac{\partial F_0}{\partial t_{-1}}|_{t_{-1}=0} \quad , \tag{3.18}$$

where F_0 is the contribution of $\mathbb{P}^1$ diagrams to the free energy of a deformed model obtained by adding the logarithmic term $t_{-1} \ln z$ to the original matrix model potential $W(z)$. As explained in [17], the planar free energy of this deformed model obeys certain Whitham-like constraints, one of which has the form:[8]

$$\frac{\partial F_0}{\partial t_{-1}} = -\int_{\mathbb{R}+i\epsilon} d\lambda \rho_0(\lambda) \log \lambda \,. \tag{3.19}$$

As discussed in [17], adding the logarithmic term $t_{-1} \ln z$ to the matrix model potential has the effect of replacing Σ with a singular algebraic curve. The latter can be mapped to a smooth Riemann surface of higher genus, which has the effect of introducing new cuts for the deformed model. This means that in the presence of the logarithmic deformation, the eigenvalues of the deformed matrix model can accumulate not only along I_i and $\tilde{I}_j$, but also along certain new loci in the complex plane (a precise description of the new cuts is given in section 3.6 of [17]). Since equation (3.18) only requires the result of (3.19) for $t_{-1} = 0$, we can in fact neglect these new cuts and evaluate the right hand side of (3.19) in the *undeformed* theory. To write the result in geometric fashion, let us consider the function:

$$\Psi(\lambda) = \int d\lambda' \rho_0(\lambda') \left[\ln|\lambda + \lambda'| + \ln|\lambda - \lambda'|\right] - W(\lambda) - W(-\lambda)\,, \tag{3.20}$$

which was introduced in section 4 of [17]. Combining (3.18), (3.19) and (3.20), we find:

$$F_1 = -\frac{s}{4}\Psi(0) - \frac{s}{2}W(0)\,, \tag{3.21}$$

where $\Psi(0)$ should be evaluated for $t_{-1} = 0$. As shown in [17], Ψ is constant along each of the cuts $\tilde{I}_j$, where its value coincides with the planar chemical potential $\tilde{\mu}_j^{(0)}$ up to an overall constant which can be fixed by choosing a cutoff $\tilde{\Lambda}$ at infinity. Upon making this choice, the chemical potentials $\tilde{\mu}_j^{(0)}$ become equal with the periods $\tilde{\Pi}_j(\tilde{\Lambda})$ computed over the B-cycles $\tilde{b}_j(\tilde{\Lambda})$ discussed at the beginning of the present section. In particular, we have $\tilde{\mu}_0 = \tilde{\Pi}_0(\tilde{\Lambda})$ and thus:

$$F_1 = -\frac{s}{4}\tilde{\Pi}_0(\tilde{\Lambda}) - \frac{s}{2}W(0)\,. \tag{3.22}$$

[8] Here we use an 'almost hermitean formulation' introduced in [17] based on the work of [36]. This amounts to requiring the eigenvalues of M to lie on the displaced real axis $\mathbb{R} + i\epsilon$, where one takes the regulator ϵ to zero at the very end of all computations. This formulation is correct only if W has even degree (since otherwise the matrix model partition function diverges). A similar relation can be written when W has odd degree, by using the general set-up of [36]. In that case, one requires the eigenvalues of M to lie on a more general contour γ in the complex plane, which must be chosen such that $\gamma \cap (-\gamma) = \emptyset$. Then one must consider the limit when γ coincides with $-\gamma$ at the end of all computations, which corresponds to working with a limiting statistical ensemble of holomorphic matrix models [36].

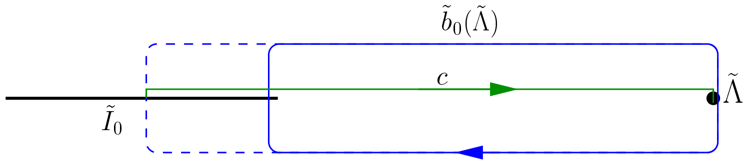

Figure 2: The orientifold contribution to the superpotential is proportional to the period $\tilde{\Pi}_0(\tilde{\Lambda})$. The relevant integral can be expressed as the integral of $u_1 - u_0$ over the curve c lying in the z-plane.

We are free to pick the cutoff $\tilde{\Lambda}$ for the $\tilde{\Pi}$-periods to be different from the cutoff Λ for the Π-periods. In particular, we can pick $\tilde{\Lambda}$ so that we absorb the contribution $-\frac{s}{2}W(0)$ to F_1. With this choice, we recover equation (3.16).

Since $\tilde{\Pi}_0 = \frac{\partial F_0}{\partial \tilde{S}_0}$, we have:

$$F_1 = -\frac{s}{4}\frac{\partial F_0}{\partial \tilde{S}_0}. \tag{3.23}$$

A similar relation was observed in [22] for the case of SO/Sp gauge theories with adjoint matter.

3.3 Comparison with the proposal of [34]

Let us compare relation (3.16) with the proposal of [34]. For this, we write (3.16) in the form:

$$F_1 = -\frac{s}{4}\int_c [u_1(z) - u_0(z)]dz\,, \tag{3.24}$$

where c is a curve in the z-plane which connects the origin with the point $\tilde{\Lambda}$ (see figure 2). To arrive at (3.24) we used analyticity of u_0 and u_1 to deform the projection of $\tilde{b}_0(\tilde{\Lambda})$ toward the origin of the z-plane as shown in figure 2 (this doesn't change the value of the integral because $u_1(x+i0) = u_0(x-i0)$ along the cut $\tilde{I}_0$).

To compare with [34], we shall express (3.24) as an integral of Ω over a 3-chain $\mathcal{C}$ in the deformed Calabi-Yau space X. Following the ideas of [2, 3, 4], this 3-chain is defined as the total space of an S^2 fibration over the curve c. To construct this fibration explicitly, we write the deformed Calabi-Yau (3.1) in the form:

$$s^2 + t^2 = (u - u_0(z))(u - u_1(z))(u - u_2(z))\,, \tag{3.25}$$

where $x = s + it$ and $y = s - it$. Let us fix the point z and consider the segment $I_{01}(z) = [u_0(z), u_1(z)]$ which connects the points $u_0(z)$ and $u_1(z)$ in the u-plane. Picking a square root $\sigma(u,z)$ of the right hand side of (3.25), we have $s^2 + t^2 = \sigma(u,z)^2$, where $\sigma(u,z)$ vanishes when u coincides with one of $u_j(z)$. Let $C(u,z)$ denote the circle $\alpha^2 + \beta^2 = 1$ obtained by requiring that $\alpha := s/\sigma(u,z)$ and $\beta := t/\sigma(u,z)$ are both real. Varying u inside the interval $I_{01}(z)$, we obtain a two-sphere S^2_z given as an S^1 fibration over this segment whose fiber collapses to a point at the ends of the interval. Finally, we fiber this S^2 over c by letting z vary along this curve. This gives the desired 3-chain $\mathcal{C}$ in the deformed Calabi-Yau. The boundary of $\mathcal{C}$ consists of the two-spheres sitting above $\tilde{\Lambda}$ and above the origin of the z-plane:

$$\partial\mathcal{C} = S^2_0 \cup S^2_{\tilde{\Lambda}}\,. \tag{3.26}$$

This boundary intersects the orientifold plane (3.4) for $x = -y \Leftrightarrow s = 0$, which gives $\alpha = s/\sigma(u, 0) = 0$ and thus $\beta = \pm 1$. This is a circle Γ inside S^2_0 traced by two opposite points of the circle $C(z, u)$ when u varies along $I_{01}(0)$. Thus:

$$(\partial \mathcal{C}) \cap O = \Gamma \,. \tag{3.27}$$

Using the arguments of [2, 3, 4], one can immediately show the relation:[9]

$$F_1 = -\frac{s}{4} \int_{\mathcal{C}} \Omega \quad , \tag{3.28}$$

which reduces to (3.24) upon performing the integral over the two-sphere fibers of $\mathcal{C}$.

According to the proposal of [34] the superpotential contribution of the orientifold should be given by integrating the holomorphic 3-form Ω along a three-chain whose boundary consists of the internal part O of the orientifold 'plane' and a piece sitting at infinity (which in our case is represented by $S^2_{\hat{\Lambda}}$ after introducing the cutoff). This is almost exactly what expression (3.28) does. However, in our case the O5 'plane' is noncompact and the 3-chain $\mathcal{C}$ which reproduces the result known from the matrix model intersects the orientifold fixed locus in X along a *circle*. We attribute this phenomenon to the fact that in our case the internal part O of the O5 'plane' is noncompact.

4. Engineering of SO(N) and Sp($N/2$) gauge theories with adjoint matter

In this section we study the geometric engineering of $\mathcal{N} = 1$ supersymmetric gauge theories with orthogonal or symplectic gauge group with the help of the orientifold action introduced in the previous sections. It is well known that the T-dual Hanany-Witten construction allows the use of either orientifold four-planes or orientifold six-planes [38] as a means to engineer SO/Sp gauge theories. In fact the type-IIB construction with orientifold five-'planes' T-dual to orientifold four-planes has already been studied in [8] and in relation to matrix models in [22]. Here we consider the case of an orientifold 6-plane, where both NS branes are rotated and/or deformed with respect to their $\mathcal{N} = 2$ position in a manner dictated by $W'(z)$.

4.1 Two geometric engineering constructions

Let us start with the singular A_1 fibration $X_{1,0}$ given by:

$$X_{1,0}: \quad xy = (u - t_0(z))(u - t_1(z)) \,, \tag{4.1}$$

where $t_0(z) = W'(z)$ and $t_1(z) = -W'(z)$. This fibration admits the two-section:

$$\Sigma_{1,0}: \quad x = y = 0 \,, \qquad (u - W'(z))(u + W'(z)) = 0 \,, \tag{4.2}$$

whose irreducible components are two rational curves.

[9] This requires an appropriate normalization of Ω.

The resolution $\hat{X}_1$ can be described globally as the complete intersection:

$$\begin{aligned}
\beta(u - t_0(z)) &= \alpha x \\
\alpha(u - t_1(z)) &= \beta y \\
(u - t_0(z))(u - t_1(z)) &= xy
\end{aligned} \tag{4.3}$$

in the ambient space $\mathbb{P}^1[\alpha, \beta] \times \mathbb{C}^4[z, u, x, y]$. The exceptional $\mathbb{P}^1$'s of the resolution sit above the singular points of $X_{1,0}$, which are determined by $x = y = u = 0$ and $z = z_j$, where z_j are the roots of W'. The resolution admits the U(1) action:

$$([\alpha, \beta], z, u, x, y) \longrightarrow ([e^{-i\theta}\alpha, \beta], z, u, e^{i\theta}x, e^{-i\theta}y)\,, \tag{4.4}$$

and T-duality with respect to its orbits allows one to recover the Hanany-Witten description.

Let us now add an orientifold. By analogy with the previous sections, we can use the action:

$$([\alpha, \beta], z, u, x, y) \longrightarrow ([-\beta, \alpha], -z, u, -y, -x)\,, \tag{4.5}$$

which is a symmetry provided that one takes $W(z)$ to be an *even* polynomial, whose degree we denote by $2n$. Then one can index the critical points of W by z_j with $j = -(n-1), \ldots, n-1$, such that $z_{-j} = -z_j$. In particular, we have the critical point $z_0 = 0$. We let D_j denote the exceptional $\mathbb{P}^1$ sitting above z_j. The action (4.5) maps D_j into D_{-j} and in particular it stabilizes the central exceptional curve D_0, on which it acts through antipodal involution. Since $W'(0) = 0$, the central fiber of $X_{1,0}$ is an A_1 singularity with equation $xy = u^2$, while the central fiber of $\hat{X}_1$ is its minimal resolution.

The action (4.5) projects to the following involution of $X_{1,0}$:

$$(z, u, x, y) \longrightarrow (-z, u, -y, -x)\,, \tag{4.6}$$

whose fixed point set is given by:

$$O_{1,0}: \quad x = -y\,, \qquad z = 0\,, \qquad x^2 + u^2 = 0\,. \tag{4.7}$$

This is a reducible curve whose two rational components $x = \pm iu$ sit in the central fiber of $X_{1,0}$. The fixed point set $\hat{O}_1$ of (4.5) is is a disjoint union of two rational curves lying inside the central fiber of the resolved space (the singular point $(u, x) = (0, 0)$ of (4.7) is replaced by the two points $(x, u, \xi) = (0, 0, \pm i)$ where $\xi = \alpha/\beta$). Thus the orientifold action (4.5) determines a (disconnected) orientifold 5-'plane' which we denote by O5.[10]

Since W is an even polynomial, the resolved space $\hat{X}_1$ admits another holomorphic involution, which acts as:

$$([\alpha, \beta], z, u, x, y) \longrightarrow ([\alpha, \beta], -z, -u, -x, -y)\,. \tag{4.8}$$

[10]Since O5 has two connected components, it should be viewed more properly as *two* orientifold fixed 'planes' whose RR charge adds to that of a single O5 plane. This unusual situation is due to the fact that we work with a nontrivial geometric background and our orientifold 'planes' are curved.

This orientifold action was used in previous studies [33, 8, 22] to geometrically engineer the SO(N) and Sp($N/2$) theories with one adjoint chiral multiplet. As we shall see below, the same theories can be engineered by using the action (4.5), and we are interested in comparing the two realizations.

The fixed point set $\hat{O}'_1$ of the action (4.8) coincides with the exceptional curve D_0 in $\hat{X}_1$ which sits above the singular point $x = y = z = u = 0$ of $X_{1,0}$. We will denote the associated orientifold 5-'plane' by O5'. Notice that O5' coincides with the worldvolume of the stack of D5-branes which is wrapped on the central $\mathbb{P}^1$. This is of particular importance for the geometric transition. After this transition, the action (4.8) will generically become fixed point free so the O5'-'plane' disappears, being replaced by the appropriate RR flux on the three-cycle creating by smoothing the double point sitting at the origin. This is quite different from the behavior of $O5$, which survives the transition as we shall see in a moment.

4.2 The T-dual configurations

To extract the T-dual Hanany-Witten configurations, we again use a local description valid on a subset $\tilde{X}_1 \subset \hat{X}_1$. In the present case, it is given by two copies U_0 and U_1 of $\mathbb{C}^3$ with coordinates (x_i, u_i, z_i) $(i = 0, 1)$ which are glued together according to:

$$(x_1, u_1, z_1) = \left(\frac{1}{u_0}, x_0 u_0^2 - 2W'(z_0)u_0, z_0\right). \tag{4.9}$$

The resolution map τ has the form:

$$\begin{aligned}(z, u, x, y) &= (z_0, x_0 u_0 - W'(z_0), x_0, u_0(x_0 u_0 - 2W'(z_0))), &(4.10)\\ &= (z_1, x_1 u_1 + W'(z_1), x_1(x_1 u_1 + 2W'(z_1)), u_1). &(4.11)\end{aligned}$$

The U(1) action (4.4) is given by:

$$(z_i, u_i, x_i) \longrightarrow (z_i, e^{-i\theta} u_i, e^{i\theta} x_i), \tag{4.12}$$

and fixes the rational curves $u_i = x_i = 0$, which are the proper transforms of the two components of (4.2).

The flat coordinates of the Hanany-Witten construction are given by:

$$x^4 + ix^5 = x_0 u_0 - W'(z_0) = x_1 u_1 + W'(z_1), \qquad x^6 = \frac{1}{2}(|x_1|^2 - |u_0|^2) \tag{4.13}$$

and $z = x^8 + ix^9$, while x^7 is the periodic coordinate along the orbits of the U(1) action (4.12).

As mentioned above, the T-dual background contains two NS5-branes $\mathcal{N}_0$ and $\mathcal{N}_1$, which sit at

$$\mathcal{N}_0: \quad x^4 + ix^5 = -W'(z), \qquad x^6 = +\infty \tag{4.14}$$

and:

$$\mathcal{N}_1: \quad x^4 + ix^5 = +W'(z), \qquad x^6 = -\infty. \tag{4.15}$$

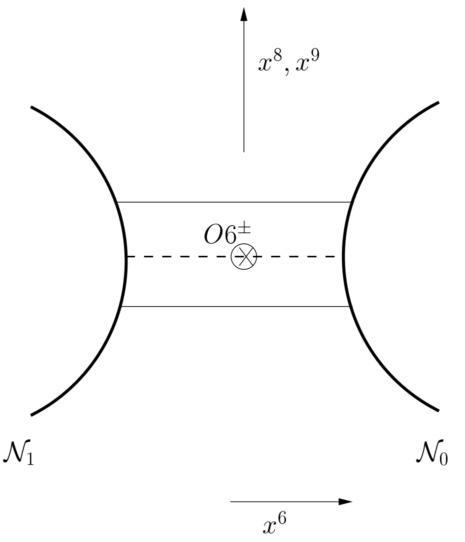

Figure 3: Brane construction with an O6 plane.

The orientifold (4.5) acts in local coordinates as:

$$(z_0, x_0, u_0) \longleftrightarrow (-z_1, -u_1, -x_1)\,. \tag{4.16}$$

This action fixes the locus $u_0^2 + 1 = z = 0$, which is a union of two disjoint rational curves. Using (4.13) we find that under T-duality this maps to an O6-plane sitting at $x^6 = x^8 = x^9 = 0$ (figure 3). Note that there is a single dual O6-plane, even though the original O5-plane in the resolved space $\hat{X}_1$ has two connected components. This is due to nonlinearity of the map (4.13).

On the other hand, the orientifold (4.8) acts in local coordinates as:

$$(z_0, x_0, u_0) \to (-z_0, -x_0, u_0) \qquad \text{in the patch } \mathrm{U}_0\,, \tag{4.17}$$

$$(z_1, x_1, u_1) \to (-z_1, x_1, -u_1) \qquad \text{in the patch } \mathrm{U}_1\,. \tag{4.18}$$

In the T-dual picture, this maps to an orientifold four-plane located at $x^4 = x^5 = x^8 = x^9 = x^7 = 0$ (figure 4).

For both choices of orientifold action, wrapping D5-branes on the exceptional divisors will give an $\mathcal{N} = 1$ gauge theory with orthogonal or symplectic gauge group depending on the RR-charge of the orientifold 5-'plane'. The theory also contains a chiral multiplet Φ in the adjoint representation of the gauge group with a tree-level superpotential $trW(\Phi)$. The classical vacua have the form:

$$\Phi = \mathrm{diag}(0_{N_0}, -\zeta_1 1_{N_1}, \zeta_1 1_{N_1}, \ldots, -\zeta_n 1_{N_n}, \zeta_n 1_{N_n}) \tag{4.19}$$

where ζ_j are the zeroes of $W'(z)$ and $N_0 + 2\sum_j N_j = N$. Here $j = -n, \ldots, n$, with $\zeta_{-j} = -\zeta_j$ and in particular $\zeta_0 = 0$. Such a vev of Φ breaks the gauge group down to the product:

$$G_0 \times \prod_{j=1}^{n} \mathrm{U}(N_j)\,, \tag{4.20}$$

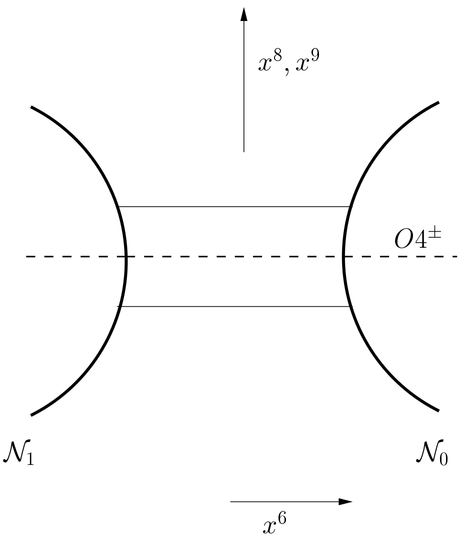

Figure 4: Brane construction with an O4 plane.

where $G_0 = \mathrm{SO}(N_0)$ or $\mathrm{Sp}(N_0/2)$ according to whether we have an orientifold plane of positive or negative charge.

4.3 Low energy descriptions after the geometric transition

After the geometric transition, one must distinguish how one computes the effective superpotential in the two constructions.

4.3.1 Engineering through an O5-'plane' T-dual to an O4-plane

Let us first review the results of [22] for the orientifold action (4.8). Since in this case the orientifold is replaced by a contribution to the RR flux during the geometric transition, its effect is to modify the flux through the three cycle $\tilde{A}_0$ of (4.21) which results by smoothing the central double point of X_1. At low energies, the SO/Sp gauge theory confines producing gaugino condensates $\tilde{S}_j$. This corresponds to a geometric transition during which the exceptional $\mathbb{P}^1$'s are blown down and the resulting singular space X_1 is smoothed to:

$$xy = u^2 - W'(z)^2 + 2f_0(z)\,. \tag{4.21}$$

As usual, the deformation must be log-normalizable and respect the orientifold projection, so $f_0(z)$ must be an even polynomial of degree $2n-2$. We also included a factor of two in order to match the normalization that arises naturally in the related matrix model.

The gaugino condensates $\tilde{S}_j$ can be identified with periods of the holomorphic three-form of (4.21) over the 3-spheres produced by the transition. Using notation similar to that of section 3, the gaugino condensate $\tilde{S}_0$ in the SO/Sp factor of the unbroken gauge group can be identified with the period of the holomorphic three-form Ω along the short invariant cycle A_0:

$$2\pi i S_0 = \frac{1}{2}\int_{A_0} \Omega\,. \tag{4.22}$$

Then the RR-flux through the short invariant cycle A_0 is:

$$\frac{1}{2}\int_{A_0} H = \frac{N_0}{2} - s\,. \tag{4.23}$$

We find the flux superpotential:

$$W_{\rm eff} = (\frac{N_0}{2} - s)\frac{\partial F_0}{\partial S_0}\,. \tag{4.24}$$

Note that there is no orientifold contribution because the orientifold fixed 'plane' is replaced by a RR flux after the transition. In the corresponding matrix model there are of course still diagrams with $\mathbb{RP}^2$ topology and as shown in [37] they contribute to the superpotential as:

$$W_{\rm eff} = \frac{N_0}{2}\frac{\partial F_0}{\partial S_0} + 4F_1\,, \tag{4.25}$$

where $F = F_0 + \frac{1}{N}F_1 + O(1/N^2)$ is the microcanonical partition function of the matrix model. This allows one to identify the subleading $1/N$ contribution to the superpotential as [22]:

$$F_1 = -\frac{s}{4}\frac{\partial F_0}{\partial S_0}\,. \tag{4.26}$$

Reducing to the deformed Riemann surface:

$$u^2 = W'(z)^2 - 2f_0(z)\,, \tag{4.27}$$

(the two-section $x = y = 0$ of the deformed Calabi-Yau (4.21)), the relevant period integrals take the form:

$$S_0 = \int_{a_0} u\frac{dz}{2\pi i}\,, \qquad \frac{\partial F_0}{\partial S_0} = \Pi(\Lambda) = \int_{b_0(\Lambda)} u\,dz\,. \tag{4.28}$$

The 'cycle' b_0 has intersection -1 with a_0 and is non-compact. The integral over b_0 has been regularized by introducing a cutoff Λ.

4.3.2 Engineering through an O5-'plane' T-dual to an O6-plane

In this case, the transition again produces the space (4.21), this time with the orientifold action (4.6), which fixes the locus $z = 0$, $x = -y$. This is the smooth rational curve:

$$x^2 + u^2 + 2f_0(0) = 0\,. \tag{4.29}$$

Note that the O5-'plane' becomes connected after the geometric transition.

To extract the effective superpotential, we shall use the trick of realizing the theory with an O5-'plane' T-dual to an O6-plane as a certain Higgs branch of the theory of with symmetric or antisymmetric matter. This branch is obtained from (2.34) when taking $W(z)$ to be even of degree $d+1 = 2n$, which forces the $d-1 = 2n-2$ critical points z_j of W to coincide with the $2\delta = 2n-2$ nonvanishing solutions $\tilde{z}_j$ of $W'(z) - W'(-z) = 0$. With an appropriate enumeration, we can then take $\tilde{z}_j = z_j^+$ for all positive j and $\tilde{z}_j = z_{-j}^-$ for all negative j. We also have the null value $\tilde{z}_0 = 0$. Then one can further Higgs by giving nonzero expectation values to Q, which forces us to keep only solutions of type (4.19), thus recovering the vacua of the SO / Sp theory.

In the brane construction, this process amounts to displacing the central NS-brane in order to give vevs to Q, which eliminates all stacks of D4 branes stretching between the middle and outer NS branes. Then the middle NS brane can be decoupled, which recovers the realization which uses only the two outer NS branes.

In geometric engineering, this corresponds to starting with the special case when the classical curve (2.3) is reduced to the form:

$$\Sigma_0^{\mathrm{special}} : \ u(u - W'(z))(u + W'(z)) = 0\,, \tag{4.30}$$

which has *triple* points at the critical points of W. Then giving a vev to Q amounts to forgetting the branch $u_2 \equiv 0$, thereby recovering the classical SO/Sp curve (4.2). Starting with such special orientifolded A_2 fibrations, the geometric transition will produce a deformed space X which is only a *partial* smoothing of X_0. Namely, the A_2 singularities of X_0 are deformed into A_1 singularities, which corresponds to partially smoothing the triple points of (4.30) by replacing the factor (4.2) with its deformation (4.27).

In the matrix model, this amounts to requiring that all filling fractions S_j must vanish, so that all cuts connecting the branches u_0 and u_2 as well as u_0 and u_2 are reduced to double points. As we shall see in a moment, this amounts to restricting to planar eigenvalue distributions which are symmetric with respect to the origin of the z-plane.

To see this explicitly, remember from [17] that the large-N spectral curve of the theory with symmetric or antisymmetric matter (and without the logarithmic deformation) has the form (3.2), where the polynomials p, q have the following expression in terms of matrix model data:

$$\begin{aligned} p(z) &= t(z)^2 + t(z)t(-z) + t(-z)^2 - f_0(z) - f_0(-z) \\ q(z) &= -t(z)t(-z)[t(z) + t(-z)] + t(z)f_0(-z) + t(-z)f_0(z) - g_0(z) - g_0(-z)\,. \end{aligned} \tag{4.31}$$

Here $f_0(z)$ and $g_0(z)$ are polynomials of degree $d-1 = 2n-2$, given explicitly by:

$$\begin{aligned} f_0(z) &= \int d\lambda \rho_0(\lambda) \frac{U'(z) - U'(\lambda)}{z - \lambda} \\ g_0(z) &= \int d\mu \int d\lambda \rho_0(\lambda) \frac{U'(z) - U'(\lambda)}{(\lambda + \mu)(z - \lambda)} \end{aligned} \tag{4.32}$$

where $U'(z) = W'(z) + \frac{s}{2N}\frac{1}{z}$.

In the case of interest for this section, we have $W'(-z) = -W'(z)$ so that $U'(-z) = -U'(z)$ and $t(-z) = -t(z) = W'(z)$. Since $degW'(z) = d = 2n-1$, we have $\delta = \left[\frac{d-1}{2}\right] = n-1$ and the surface (3.2) has $2d = 2(n-1)$ cuts of type I_j and $2\delta + 1 = 2n-1$ cuts of type $\tilde{I}_j$.

Let us assume that $\rho_0(-\lambda) = \rho_0(\lambda)$ for all λ. Then equations (4.32) immediately imply that $f_0(-z) = f_0(z)$ and $g_0(-z) = -g_0(z)$, so that (4.31) give:

$$\begin{aligned} p(z) &= W'(z)^2 - 2f_0(z) \\ q(z) &= 0\,. \end{aligned} \tag{4.33}$$

Therefore, the spectral curve (3.2) reduces to:

$$\Sigma_{\text{special}}: \quad u^3-(W'(z)^2-2f_0(z))u=0 \Longleftrightarrow u(u-W'(z)^2-2f_0(z))=0 \qquad \text{with } f_0 = \text{even}\,. \tag{4.34}$$

Conversely, let us assume that (3.2) has the form (4.34). Then $u_2 \equiv 0$ and $u_1(z) = u_0(-z) = -u_0(z)$, which means that all cuts I_j are reduced to ordinary double points sitting at the endpoints of the cuts $\tilde{I}_j$ (there are two such double points for each cut $\tilde{I}_j$). These double points correspond to the zeroes of the degree $2(2n-1)$ even polynomial $W'(z)^2 - 2f_0(z)$, which are distributed symmetrically with respect to the origin of the z-plane.

Now remember from section 3.5.2 of [17] (see equation (3.81) of that paper) that the planar spectral density $\rho_0(\lambda)$ is symmetric along the union of all cuts of type $\tilde{I}_j$. Since in our case there are no other cuts, the support of ρ_0 coincides with $\cup_j \tilde{I}_j$ and it immediately follows that $\rho_0(-\lambda) = \rho_0(\lambda)$ for all λ. Thus:

The large-N spectral density $\rho_0(\lambda)$ of the matrix model for symmetric or antisymmetric matter with an even tree-level superpotential W of degree $2n$ is symmetric if and only if the spectral curve has the form (4.34), i.e. iff. the polynomials $p(z)$ and $q(z)$ have the form (4.33), where $f_0(z)$ is an even *polynomial of degree $2n-2$.*

Hence the field theory higgsing described above translates into imposing a symmetric distribution for $\rho_0(\lambda) = \lim_{N\to\infty}\langle\rho(\lambda)\rangle$. In this case the spectral curve reduces to the form (4.34), which coincides (up to the spectator branch $u \equiv 0$) with the spectral curve (4.27) of the SO/Sp model. It follows that the planar free energy on this branch of the moduli space of filling fractions of the model with symmetric/antisymmetric matter must agree with that of the SO/Sp model, provided that one identifies the filling fractions $\tilde{S}_j$ of the former with those of the latter. One the other hand, the $\mathbb{RP}^2$ contributions are given by relations (3.23) and (4.26), which have the same form and determine F_1 in terms of F_0. It follows that the subleading terms F_1 must also agree.

Observation. If one formally restricts to exactly symmetric distributions of eigenvalues of the model with symmetric or antisymmetric matter, then one has the 'quantum' relation $\omega(z) = -\omega(-z)$. Combining this with $U'(z) = -U'(-z)$, the quadratic loop equation in (3.17) reduces to:

$$\frac{1}{2}\langle\omega(z)^2\rangle = \int_\gamma \frac{dx}{2\pi i}\frac{U'(x)}{z-x}\langle\omega(x)\rangle \tag{4.35}$$

while the cubic loop equation becomes a tautology. To arrive at (4.35), we used the identity:

$$\frac{1}{z}\left(\frac{1}{z-\lambda_i}+\frac{1}{z+\lambda_i}\right) = \frac{1}{\lambda_i}\left(\frac{1}{z-\lambda_i}-\frac{1}{z+\lambda_i}\right) = \frac{2}{z^2-\lambda^2}\,. \tag{4.36}$$

Since the eigenvalues are distributed symmetrically, the resolvent takes the form $\omega(z) = \sum_i \frac{2z}{z^2-\lambda_i^2}$ [22]. Using $U'(x) = W'(x) + \frac{s}{2N}\frac{1}{x}$, we can write (4.35) as:

$$\frac{1}{2}\langle\omega(z)^2\rangle - \frac{s}{2N}\frac{1}{z}\langle\omega(z)\rangle = \int_\gamma \frac{dx}{2\pi i}\frac{W'(x)}{z-x}\langle\omega(x)\rangle\,. \tag{4.37}$$

This coincides with the exact loop equations for the SO/Sp theory with adjoint matter, as derived in [39, 22].

Notice however that such a symmetric distribution of eigenvalues might be unstable to orders $O(1/N^2)$, and a priori there is no meaningful way to impose the symmetry of eigenvalues at the quantum level in the matrix model (as opposed to symmetry of their (large-N) averaged distribution $\langle\rho(\lambda)\rangle$, which is all we required before). Indeed, one expects that gravitational F-terms in the effective field theory should distinguish between the two realizations. In the matrix model, such gravitational corrections will correspond [23] to contributions of order $O(1/N^2)$. Therefore we expect that the two backgrounds obtained after the large-N transition differ through F-terms of order $O(1/N^2)$ or higher.

5. Conclusions

We investigated the geometric engineering of $\mathcal{N}=1$ gauge theory with gauge group U(N) and matter in the adjoint and symmetric or antisymmetric representations. We showed that such theories can be realized as certain orientifolds of resolved Calabi-Yau A_2 fibrations, where one wraps $D5$-branes on the exceptional $\mathbb{P}^1$'s of the resolution. The orientifold action one has to consider defines an orientifold 5-plane whose internal part coincides with a *noncompact* rational curve sitting in the resolved Calabi-Yau space.

We also gave the explicit relation of this construction with the Hanany-Witten realization [15, 16] through orientifolded brane configurations in IIA string theory. This is implemented by T-duality with respect to the orbits of a certain U(1) action on the resolved Calabi-Yau, along the lines of [7, 12, 13]. Upon giving an explicit construction of the T-dual coordinates, we showed that the orientifold 5-'plane' used in our geometric engineering maps to the O6-plane used in [15, 16].

Following the ideas of [2, 3, 4], we considered the geometric transition which replaces the resolved Calabi-Yau space with its deformation. Upon restricting to deformations compatible with the geometric symmetry used in the orientifold construction, we found that the orientifold 5-'plane' *survives* the geometric transition, and thus contributes to the effective superpotential of the resulting background. This gives the geometric explanation of the $\mathbb{RP}^2$ diagram contributions found in [17], which affect the field theory glueball superpotential after confinement.

Using the matrix model results of [17], we gave a geometric expression for this orientifold contribution as an integral over a certain 3-chain in the deformed Calabi-Yau space, and compared with the proposal of [34].

We also discussed the Higgs branch of our theories obtained by giving a vev to the symmetric or antisymmetric tensor, and showed that this Higgs branch recovers the SO(N) or Sp($N/2$) gauge theory with adjoint matter. Namely, we showed that this process leads to a geometric engineering of such theories which is T-dual to their realization obtained by adding an O6-plane to a Hanany-Witten brane configuration. Using the matrix model results of [17], we extracted the glueball superpotential on this branch, and showed that it recovers the results of [22]. The later where obtained in [22] though a different geometric engineering (which is T-dual to a Hanany-Witten construction involving an O4-plane). After the geometric transition, this alternate construction leads to a pure flux background, in which the O5-'plane' is replaced by R-R fluxes. This implies matching of $\mathbb{RP}^2$ contributions to the superpotential between the two constructions, which we checked explicitly by using the matrix model results of [17] and [22].

One can apply similar methods to other models which admit a geometric engineering. An interesting example of this kind is the chiral $\mathrm{U}(N)$ model with adjoint as well as symmetric, antisymmetric and fundamental matter which is discussed in [32].

Acknowledgments

This work was supported by DFG grant KL1070/2-1. R. T. was supported in part by the DOE Contract DE-AC03-76SF00098 and in part by the NSF grant PHY-0098840.

A. Geometric engineering without a tree-level superpotential

In this appendix we give a geometric argument which explains why the orientifold projection of our IIB background leads to (anti)symmetric matter. For this, one can consider the limit when the tree-level superpotential vanishes and the theory acquires $\mathcal{N}=2$ supersymmetry. In this limit, the Calabi-Yau space X_0 becomes the trivial A_2 fibration $\mathbb{C}\times X_0(0)$, where the fiber $X_0(0)$ is an A_2 singularity.

More generally, the relevant geometry can be obtained by considering the non-generic case when $W'(0)=0$ (note that this case was explicitly excluded in section 2). Then the central fiber $X_0(0)$ becomes an A_2 singularity $xy=u^3$, while $\hat{X}(0)$ becomes its minimal resolution:

$$\begin{aligned}
\xi_1 x &= u \\
\xi_2 u &= y \\
\frac{\xi_2}{\xi_1} &= u \\
xy &= u^3 \,.
\end{aligned} \tag{A.1}$$

Outside the locus $x=y=u=0$, (A.1) determines[11] $\xi_1=\frac{u}{x}=\frac{y}{u^2}$ and $\xi_2=\frac{y}{u}=\frac{u^2}{x}$. For $x=y=u=0$, we are left with the constraint $\frac{\xi_2}{\xi_1}=0$, which gives the two exceptional $\mathbb{P}^1$'s with equations $\xi_1=\infty$ and $\xi_2=0$. The orientifold action on $\hat{X}$ fixes the locus $\hat{O}: x=-y, \xi_1\xi_2=-1$ in $\hat{X}(0)$, which is a a smooth rational curve passing through the common point p of the two exceptional $\mathbb{P}^1$'s:

$$D_0^{(1)}\cap D_0^{(2)}=\{p\}: x=y=0\,, \qquad \xi_1=\infty\,, \qquad \xi_2=0\,. \tag{A.2}$$

The curve $\hat{O}$ intersects the exceptional fibers only at this point, and the orientifold action maps $D_0^{(1)}$ into $D_0^{(2)}$ according to relation (2.24). Finally, note that $\hat{O}$ projects to the fixed locus O_0 of the action (2.23) on X_0, which in the case $W'(0)=0$ is the cuspidal curve:

$$x=-y\,, \qquad x^2+u^3=0\,. \tag{A.3}$$

[11] When $xy=0$, but $x\neq y$, one uses one of the forms given in the text to remove the ambiguity. For example, let $x=0$ and $y\neq 0$. Then one must use the forms $\xi_1=\frac{y}{u^2}$ and $\xi_2=\frac{y}{u}$ in order to determine ξ_1 unambiguously as a point in the one-point compactification $\overline{\mathbb{C}}=\mathbb{P}^1$.

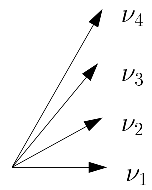

Figure 5: The toric generators $\nu_1, \ldots, \nu_4$.

A clearer description of the resolved central fiber $\hat{X}(0)$ presents it as the toric resolution $(\mathbb{C}^4 - Z)/(\mathbb{C}^*)^2$ of the A_2 singularity $X_0(0)$, with charge matrix:

$$Q = \begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \tag{A.4}$$

and toric generators given by the columns of the matrix:

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}. \tag{A.5}$$

The exceptional set is $Z = \{x_1 = x_3 = 0\} \cup \{x_2 = x_4 = 0\} \cup \{x_1 = x_4 = 0\}$.

The generators correspond to homogeneous coordinates which we denote by $x_1, \ldots, x_4$. We let $D_j = (x_j)$ be the toric divisors. Then $D_2 = D_0^{(1)}$ and $D_3 = D_0^{(2)}$ are the exceptional $\mathbb{P}^1$'s, while D_1 and D_4 are non-compact.

In the symplectic quotient description, we have the reduction of $\mathbb{C}^4$ with respect to the $\mathrm{U}(1)^2$ action defined by (A.4) with moment map equations:

$$\begin{aligned} |x_1|^2 - 2|x_2|^2 + |x_3|^2 &= \zeta_1 \\ |x_2|^2 - 2|x_3|^2 + |x_4|^2 &= \zeta_2 \,, \end{aligned} \tag{A.6}$$

where ζ_j are some positive levels. Setting $\zeta_1 = \zeta_2 = 0$ gives the A_2 singularity $X_0(0)$, which we describe in terms of invariants:

$$\begin{aligned} x &= x_1^3 x_2^2 x_3 \\ y &= x_2 x_3^2 x_4^3 \\ u &= x_1 x_2 x_3 x_4 \,, \end{aligned} \tag{A.7}$$

subject to the relation $xy = u^3$.

The orientifold action on $\hat{X}(0)$ takes the form:[12]

$$x_1 \longleftrightarrow x_4 \,, \qquad x_2 \longleftrightarrow -x_3 \,. \tag{A.8}$$

The fixed point set $\hat{O}$ is given by the smooth curve $x_1^3 x_2 = -x_3 x_4^3$. One easily checks the intersections:

$$D_2 \cap D_3 = D_2 \cap \hat{O} = D_3 \cap \hat{O} = D_2 \cap D_3 \cap \hat{O} = \{[1,0,0,1]\}\,. \tag{A.9}$$

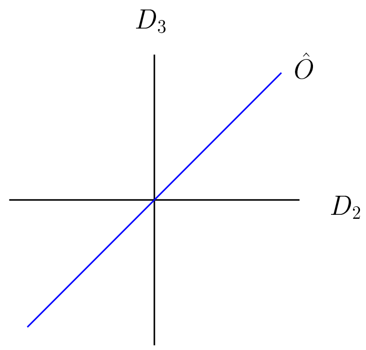

Figure 6: Orientifold action on the resolution of the A_2 singularity.

The curve $\hat{O}$ is smooth for positive ζ and degenerates to the cusp O_0 of (A.3) when $\zeta = 0$. As expected, the action (A.8) permutes the compact divisors $D_2 = D_0^{(1)}$ and $D_3 = D_0^{(2)}$. The local geometry is sketched in figure 6.

It is now trivial to find the orientifolded matter content. In the limit $W'(z) = 0$, we start by wrapping two D5-branes on the exceptional $\mathbb{P}^1$'s $D_0^{(1)}$ and $D_0^{(2)}$, endowed with trivial and isomorphic Chan-Paton bundles E_1 and E_2 of rank N. This generates chiral multiplets $Q_{21} \in \mathrm{Hom}(E_1|_p, E_2|_p) \approx \mathrm{Mat}(N, \mathbb{C})$ and $Q_{12} \in \mathrm{Hom}(E_2|_p, E_1|_p) \approx \mathrm{Mat}(N, \mathbb{C})$ from the intersection point p, as well as gauge multiplets from the strings ending on a given brane. In $\mathcal{N} = 1$ superfield language the strings ending on a brane give rise to vector fields and chiral multiplets in the adjoint representation. More precisely we have $\mathcal{W}_{\alpha.j} \in \mathrm{Aut}(E_{j,p}) \approx \mathrm{Mat}(N, \mathbb{C})$ and $\Phi_j \in \mathrm{Aut}(E_{j,p}) \approx \mathrm{Mat}(N, \mathbb{C})$ for $j \in \{1, 2\}$, where Φ_j correspond to moving the brane along the base direction z in the trivial fibration $\mathbb{C} \times X(0)$.

The result is an A_2 quiver field theory whose node potentials W_1 and W_2 arise when one deforms the trivial fibration $\mathbb{C} \times \hat{X}(0)$ in order to obtain the nontrivial fibration $\hat{X}$ over the z-plane.

Let us now add the orientifold 'plane'. Since the orientifold action permutes the two $\mathbb{P}^1$'s, the projection must relate each of Q_{12} and Q_{21} to its transpose, up to a similarity defined by homomorphisms $\gamma_1 \in \mathrm{Hom}(E_1|_p, E_2|_p)$ and $\gamma_2 \in \mathrm{Hom}(E_2|_p, E_1|_p)$:

$$Q_{ij} \to \gamma_i Q_{ij}^T \gamma_j^{-1} \tag{A.10}$$

Since this action must square to the identity, we have the constraints:

$$\gamma_2 = \pm \gamma_1^T \, . \tag{A.11}$$

To recover our theories, we choose $\gamma_1 = +1_N$ and $\gamma_2 = \pm 1_N$, which give respectively the projections:

$$Q_{ij}^T = +Q_{ij} \qquad \text{or} \qquad Q_{ij}^T = -Q_{ij} \, . \tag{A.12}$$

These are precisely the projections used in the introduction, corresponding to the two choices $s = \pm 1$. Finally notice that on the vector multiplets and the chiral multiplets in

[12] In the symplectic quotient description, this is a symmetry if we set $\zeta_1 = \zeta_2 = \zeta$.

the adjoint representation the orientifold projection acts as

$$\mathcal{W}_{\alpha,1} \to -\gamma_1 \mathcal{W}_{\alpha,2}^T \gamma_1^{-1}\,, \tag{A.13}$$
$$\mathcal{W}_{\alpha,2} \to -\gamma_2 \mathcal{W}_{\alpha,1}^T \gamma_2^{-1}\,, \tag{A.14}$$
$$\Phi_1 \to -\gamma_1 \Phi_2^T \gamma_1^{-1}\,, \tag{A.15}$$
$$\Phi_2 \to -\gamma_2 \Phi_1^T \gamma_2^{-1}\,. \tag{A.16}$$

The additional sign arises for the vector fields because the vertex operators of vectors are odd under worldsheet parity, whereas the minus sign for the chiral multiplets has its origin in the geometric action of the orientifold. According to these projections the two factor groups of the A_2 quiver gauge theory are identified as $U_1 = (U_2^T)^{-1} = U_2^*$ for both choices of γ_i. Writing now $\Phi = \Phi_1 = -\Phi_2^T$, $Q = Q_{21}$ and $\tilde{Q} = Q_{12}$ we can compute the projected superpotential (2.32)

$$W = \mathrm{tr}(Q\Phi\tilde{Q}) - \mathrm{tr}(\tilde{Q}(-\Phi^T)Q) = 2\ \mathrm{tr}(Q\Phi\tilde{Q})\,. \tag{A.17}$$

The factor 2 can be absorbed in the normalization of Q and $\tilde{Q}$ to produce (1.4).

References

[1] C. Vafa, *Superstrings and topological strings at large-N*, *J. Math. Phys.* **42** (2001) 2798 [hep-th/0008142].

[2] F. Cachazo, K.A. Intriligator and C. Vafa, *A large-N duality via a geometric transition*, *Nucl. Phys.* **B 603** (2001) 3 [hep-th/0103067].

[3] F. Cachazo, S. Katz and C. Vafa, *Geometric transitions and $N = 1$ quiver theories*, hep-th/0108120.

[4] F. Cachazo, B. Fiol, K.A. Intriligator, S. Katz and C. Vafa, *A geometric unification of dualities*, *Nucl. Phys.* **B 628** (2002) 3 [hep-th/0110028].

[5] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152].

[6] K. Landsteiner and C.I. Lazaroiu, *Geometric regularizations and dual conifold transitions*, *J. High Energy Phys.* **04** (2003) 028 [hep-th/0303054].

[7] K.-h. Oh and R. Tatar, *Duality and confinement in $N = 1$ supersymmetric theories from geometric transitions*, *Adv. Theor. Math. Phys.* **6** (2003) 141 [hep-th/0112040].

[8] J.D. Edelstein, K. Oh and R. Tatar, *Orientifold, geometric transition and large-N duality for* SO / Sp *gauge theories*, *J. High Energy Phys.* **05** (2001) 009 [hep-th/0104037].

[9] H. Fuji and Y. Ookouchi, *Confining phase superpotentials for* SO / Sp *gauge theories via geometric transition*, *J. High Energy Phys.* **02** (2003) 028 [hep-th/0205301].

[10] R. Roiban, R. Tatar and J. Walcher, *Massless flavor in geometry and matrix models*, *Nucl. Phys.* **B 665** (2003) 211 [hep-th/0301217].

[11] I. Bena, R. Roiban and R. Tatar, *Baryons, boundaries and matrix models*, hep-th/0211271; I. Bena, H. Murayama, R. Roiban and R. Tatar, *Matrix model description of baryonic deformations*, *J. High Energy Phys.* **05** (2003) 049 [hep-th/0303115].

JHEP11(2003)044

[12] K. Dasgupta, K. Oh and R. Tatar, *Geometric transition, large-N dualities and mqcd dynamics*, *Nucl. Phys.* **B 610** (2001) 331 [hep-th/0105066].

[13] K. Dasgupta, K. Oh and R. Tatar, *Open/closed string dualities and Seiberg duality from geometric transitions in M-theory*, *J. High Energy Phys.* **08** (2002) 026 [hep-th/0106040]; K. Dasgupta, K.-h. Oh, J. Park and R. Tatar, *Geometric transition versus cascading solution*, *J. High Energy Phys.* **01** (2002) 031 [hep-th/0110050].

[14] E. Witten, *Solutions of four-dimensional field theories via M-theory*, *Nucl. Phys.* **B 500** (1997) 3 [hep-th/9703166].

[15] K. Landsteiner and E. Lopez, *New curves from branes*, *Nucl. Phys.* **B 516** (1998) 273 [hep-th/9708118].

[16] K. Landsteiner, E. Lopez and D.A. Lowe, *Supersymmetric gauge theories from branes and orientifold six-planes*, *J. High Energy Phys.* **07** (1998) 011 [hep-th/9805158].

[17] A. Klemm, K. Landsteiner, C.I. Lazaroiu and I. Runkel, *Constructing gauge theory geometries from matrix models*, *J. High Energy Phys.* **05** (2003) 066 [hep-th/0303032].

[18] S.G. Naculich, H.J. Schnitzer and N. Wyllard, *Cubic curves from matrix models and generalized konishi anomalies*, *J. High Energy Phys.* **08** (2003) 021 [hep-th/0303268].

[19] S.G. Naculich, H.J. Schnitzer and N. Wyllard, *Matrix-model description of $N = 2$ gauge theories with non- hyperelliptic Seiberg-Witten curves*, *Nucl. Phys.* **B 674** (2003) 37 [hep-th/0305263].

[20] A. Hanany and E. Witten, *Type-IIB superstrings, BPS monopoles and three-dimensional gauge dynamics*, *Nucl. Phys.* **B 492** (1997) 152 [hep-th/9611230].

[21] K. Landsteiner, E. Lopez and D.A. Lowe, *$N = 2$ supersymmetric gauge theories, branes and orientifolds*, *Nucl. Phys.* **B 507** (1997) 197 [hep-th/9705199].

[22] S.K. Ashok, R. Corrado, N. Halmagyi, K.D. Kennaway and C. Romelsberger, *Unoriented strings, loop equations and $N = 1$ superpotentials from matrix models*, *Phys. Rev.* **D 67** (2003) 086004 [hep-th/0211291].

[23] R. Dijkgraaf and C. Vafa, *Matrix models, topological strings and supersymmetric gauge theories*, *Nucl. Phys.* **B 644** (2002) 3 [hep-th/0206255].

[24] R. Dijkgraaf and C. Vafa, *On geometry and matrix models*, *Nucl. Phys.* **B 644** (2002) 21 [hep-th/0207106].

[25] R. Dijkgraaf and C. Vafa, *A perturbative window into non-perturbative physics*, hep-th/0208048.

[26] F. Cachazo, M.R. Douglas, N. Seiberg and E. Witten, *Chiral rings and anomalies in supersymmetric gauge theory*, *J. High Energy Phys.* **12** (2002) 071 [hep-th/0211170].

[27] B. Acharya and E. Witten, *Chiral fermions from manifolds of G_2 holonomy*, hep-th/0109152.

[28] T.H. Buscher, *A symmetry of the string background field equations*, *Phys. Lett.* **B 194** (1987) 59.

[29] K. Landsteiner, E. Lopez and D.A. Lowe, *Duality of chiral $N = 1$ supersymmetric gauge theories via branes*, *J. High Energy Phys.* **02** (1998) 007 [hep-th/9801002].

[30] I. Brunner, A. Hanany, A. Karch and D. Lüst, *Brane dynamics and chiral non-chiral transitions*, *Nucl. Phys.* **B 528** (1998) 197 [hep-th/9801017].

[31] S. Elitzur, A. Giveon, D. Kutasov and D. Tsabar, *Branes, orientifolds and chiral gauge theories*, *Nucl. Phys.* **B 524** (1998) 251 [hep-th/9801020].

[32] K. Landsteiner, C.I. Lazaroiu, R. Tatar, *Anomaly-free chiral field theories from geometric engineering and matrix models*, in preparation.

[33] S. Sinha and C. Vafa, SO *and* Sp *Chern-Simons at large-N*, hep-th/0012136.

[34] B. Acharya, M. Aganagic, K. Hori and C. Vafa, *Orientifolds, mirror symmetry and superpotentials*, hep-th/0202208.

[35] V. Arnold, A. Gusein-Zade and A. Varchenko, *Singularites of differential maps*, Birkhäuser 1985.

[36] C.I. Lazaroiu, *Holomorphic matrix models*, *J. High Energy Phys.* **05** (2003) 044 [hep-th/0303008].

[37] H. Ita, H. Nieder and Y. Oz, *Perturbative computation of glueball superpotentials for* SO(N) *and* USp(N), *J. High Energy Phys.* **01** (2003) 018 [hep-th/0211261].

[38] A. Giveon and D. Kutasov, *Brane dynamics and gauge theory*, *Rev. Mod. Phys.* **71** (1999) 983 [hep-th/9802067].

[39] R.A. Janik and N.A. Obers, SO(N) *superpotential, Seiberg-Witten curves and loop equations*, *Phys. Lett.* **B 553** (2003) 309 [hep-th/0212069].